Practice: Binomial + Geometric Distributions Problems Key

① X = # passangers that prefer to sleep — B ✓ I ✓ N ✓ S ✓

a) $P(X=12) = \binom{25}{12}(.8)^{12}(.2)^{13} = .000293$

-The probability that exactly 12 passangers will prefer to sleep.

b) $P(X=25) = \binom{25}{25}(.8)^{25}(.2)^{0} = .00378$

- The probability that all 25 passangers prefer to sleep

c) $P(X \geq 20) = 1 - P(X \leq 19) = 1 - .38331 = .61669$

The probability that 20 or more passangers prefer to sleep.

d) $\mu_X = (n)(p) = (25)(.8) = 20$

It is expected that about 20 passangers will prefer to sleep

$\sigma_X = \sqrt{(n)(p)(1-p)} = \sqrt{(25)(.8)(.2)} = 2$

The amount of passangers that prefer to sleep vary by 2 passangers

② a) $P(X=2) = (.9)^1(.1)^1 = .09$ — B ✓ I ✓ T ✓ S ✓

The probability it take Sophie 2 tosses to catch the ball.

b) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - .19 = .81$

The probability it will take Sophie 3 or more tries to catch the ball

c) $E(X) = \frac{1}{p} = \frac{1}{.1} = 10$

$\sigma_X = \sqrt{\frac{(1-p)}{p^2}} = \sqrt{\frac{.9}{(.1)^2}} = .9487$

③ a) $E(X) = (.2)(100) = 20$

b) $\sigma_X = \sqrt{(.2)(100)(.8)} = 4$

→ The amount of correctly guessed questions can vary by ±4.

c) $p(X \geq 50) = 1 - p(X \leq 49) = 1 - 1 = 0$

B ✓
I ✓
N ✓
S ✓

④ Skip

⑤ a) $P(X \leq 2) = .0975$

b) $P(X=4) = .0429$

c) $P(X > 4) = 1 - P(X \leq 3) = 1 - .1426 = .8574$

B
I
T
S

⑥ $np \geq 10$ and $n(1-p) \geq 10$

$(2000)(.1) = 200 \geq 10$ and $(2000)(.9) = 1800 \geq 10$ ✓

B ✓
I ✓
N ✓
S ✓

$\mu_X = np = 200$ $\sigma_X = \sqrt{(2000)(.1)(.9)} = 13.416$

• $p(X \geq 225) = 1 - p(X \leq 225) = \boxed{.03120}$

• $p(X \geq 225) = 1 - p(X \leq 224) = 1 - \left(\binom{2000}{225}(.1)^{225}(.9)^{1775}\right) = \boxed{.03031}$

The actual binomial calculation is slightly lower than the normal approximation

Test 8C AP Statistics Name: key

Directions: *Work on these sheets. A random digit table is attached.*

Part 1: Multiple Choice. *Circle the letter corresponding to the best answer.*

1. It has been estimated that about 30% of frozen chickens contain enough salmonella bacteria to cause illness if improperly cooked. A consumer purchases 12 frozen chickens. What is the probability that the consumer will have more than 6 contaminated chickens?

D

(a) 0.961
(b) 0.118
(c) 0.882
(d) 0.039
(e) 0.079

$p(x>6)$
$1-p(x\le 6) = .0386$

2. The probability that a certain machine will produce a defective item is 0.20. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?

E

(a) 0.0001
(b) 0.0154
(c) 0.0015
(d) 0.2458
(e) 0.0016

$p(x\ge 5)$
$1-p(x\le 4) = .0016$

3. A professional basketball player sinks 80% of his foul shots, in the long run. If he gets 100 tries during a season, then the probability that he sinks between 75 and 90 shots (inclusive) is approximately equal to:

A

(a) Pr (-1.25 <= Z <= 2.5)
(b) Pr (-1.125 <= Z <= 2.625)
(c) Pr (-1.125 <= Z <= 2.375)
(d) Pr (-1.375 <= Z <= 2.375)
(e) Pr (-1.375 <= Z <= 2.625)

$np = (.8)(100) = 80$
$\sqrt{np(1-p)} = \sqrt{(.8)(100)(.2)} = 4$
$\frac{75-80}{4} \le x \le \frac{90-80}{4} \Rightarrow -1.25 \le z \le 2.5$

4. If X has a binomial distribution with $n = 400$ and $p = .4$, the approximate probability of the event $\{155 < X < 175\}$ is:

B

(a) 0.6552
(b) 0.6429
(c) 0.6078
(d) 0.6201
(e) 0.6320

$p(x<175) - p(x<155) = p(X\le 174) - p(X\le 154)$
$.9301 - .2881 = .642$

5. If in the previous question we change the interval to 155 <= X <= 175, the approximate probability is;

B

(a) 0.4
(b) Larger than that in the previous question
(c) Smaller than that in the previous question
(d) Equal to that in the previous question
(e) May be smaller or larger than that in the previous question

$p(x\le 175) - p(X\le 155) = .6186$

6. A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let X denote the number in the sample that say they support the increase. Suppose that 40% of *all* adults in Ohio support the increase. The probability that X is more than 650 is

Can also approximate to Normal

(a) less than 0.0001.
(b) less than 0.001.
(c) less than 0.01.
(d) 0.9960.
(e) none of these.

C

$p(X > 650)$

$1-p(x \leq 649) = .0047$

$np \geq 10$

$n(1-p) \geq 10$

7. A fair coin (one for which both the probability of heads and the probability of tails are 0.5) is tossed six times. The probability that *less* than 1/3 of the tosses are heads is

(a) 0.344.
(b) 0.33.
(c) 0.109.
(d) 0.09.
(e) 0.0043.

C

$p(x < 2) \Rightarrow p(X \leq 1) = .1094$

8. Suppose we select an SRS of size $n = 100$ from a large population having proportion p of successes. Let X be the number of successes in the sample. For which value of p would it be safe to assume the sampling distribution of X is approximately normal?

(a) 0.01
(b) 1/9
(c) 0.975
(d) 0.9999
(e) All of these.

B

$np \geq 10$

$(100)(p) \geq 10$

$p \geq .1$

9. Suppose we roll a fair die ten times. The probability that an even number occurs *exactly* the same number of times as an odd number on the ten rolls is

(a) 0.1667.
(b) 0.2461.
(c) 0.3125.
(d) 0.5000.
(e) None of these.

B

- Rolling 10 times w/ same number even # odds means it has to be 5 rolls each
- $P(X=5) = .2461$

$B \to$ mistake
$I \to \checkmark$
$N = 10000$
$S = .08$

Part 2: Free Response

Answer completely, but be concise. Write sequentially and show all steps.

10. The Internal Revenue Service estimates that 8% of all taxpayers filling out long forms make mistakes. Suppose that a random sample of 10,000 forms is selected. What is the approximate probability that more than 800 forms have mistakes?

Normal approximation to the binomial

$n = 10000$ $\quad (10000)(.08) = 800 \geq 10$ ✓

$p = .08$ $\quad (10000)(.92) = 9200 \geq 10$ ✓

$\mu_x = np = 800$

$\sigma_x = \sqrt{(n)(p)(1-p)} = \sqrt{(10000)(.08)(.92)} = 27.13$

$N(800, 27.13)$

$p(X > 800) = 1 - p(X \leq 800) =$

$1 - p(z \leq 0) =$

$1 - .5 = .5$

Binomial

$P(X > 800) = 1 - P(X \leq 800) = 1 - .5094 = \boxed{.4906}$

There is a $\boxed{.5}$ probability that more than 800 forms have mistakes.

11. A survey conducted by the Harris polling organization discovered that 63% of all Americans are overweight. Suppose that a number of randomly selected Americans are weighed.

(a) Find the probability that the fourth person weighed is the first person to be overweight.

$B \to$ overweight?
$I \to \checkmark$
$T \to \checkmark$
$S = .63$
Geometric

$p(X=4) = (.37)^3(.63) = .0319$

The probability that the first person that will be overweight is the 4th person is .0319.

(b) Find the probability that it takes more than 4 people to observe the first overweight person.

$P(X > 4) = 1 - P(X \leq 4) = 1 - .9813 = .0187$

The probability that it will take more than 4 people to find the first overweight person is .0187.

(c) Find the mean and variance of the number of Americans that would have to be weighed in order to find the first person who was overweight.

$\mu_x = \frac{1}{p} = \frac{1}{.63} = 1.587$

variance $= \frac{(1-p)}{p^2} = \frac{(.37)}{(.63)^2} = .9322$

Don't need for the test

12. Amarillo Slim, a professional dart player, has an 80% chance of hitting the bullseye on a dartboard with any throw. Suppose that he throws 10 darts, one at a time, at the dartboard.

B → hit or miss
I → ✓
N → 10
S → .8

(a) Find the probability that Slim hits the bullseye exactly six times.

$p(X=6) = \binom{10}{6}(.8)^6(.2)^4 = .0881$

The probability that he will hit the target 6 times is .0881

(b) Find the probability that he hits the bullseye at least four times.

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - (\binom{10}{3}(.8)^3(.2)^7) = 1 - .0356 = .9644$

The probability that he will hit the target more than 4 times is .9644.

(c) Compute the mean and variance of the number of bullseyes in 10 throws.

$\mu_X = np = (10)(.8) = 8$

$\sigma_X = \sqrt{(n)(p)(1-p)} = 1.265$

(d) Find the probability that Slim's first bullseye occurs on the fourth throw.

$p(Y=4) = (.2)^3(.4) = .0064$

The probability that Slim first bullseye occurs on the fourth throw is .0064.

B → hit or miss
I → independent
T → ?
S → .8

(e) Find the probability that it takes Amarillo more than 2 throws to hit the bullseye.

$P(Y>2) = 1 - P(Y \leq 2) = 1 - .96 = .04$

The probability that it takes Slim more than 2 throws is .04.

13. Harlan comes to class one day, totally unprepared for a pop quiz consisting of ten multiple-choice questions. Each question has five answer choices, and Harlan answers each question randomly.

(a) Find the probability that Harlan guesses more answers correctly than would be expected by chance. $n=10$ $p=.2$

B → correct!
I → ✓
N → 10
S → .2

$E(X) = (n)(p) = (10)(.2) = 2$

$p(X>2) = 1 - p(X \leq 2) = 1 - .6778 = .3222$

The probability that Harlan guesses more than 2 (expected value) correct is .3222.

(b) Find the probability that Harlan's first correct answer occurs on or after the fourth question.

$P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - .488 = .512$

B ✓
T ✓
T ✓
S ✓

The probability that Harlan's first correct answer occurs on or after the fourth question is .512.

I pledge that I have neither given nor received aid on this test. ______________________