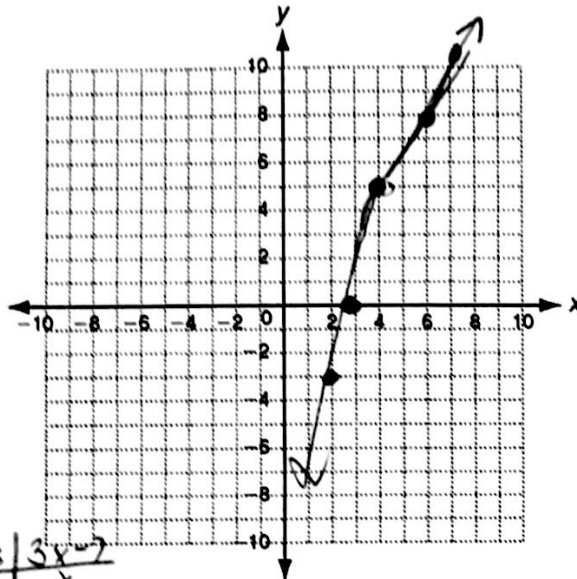


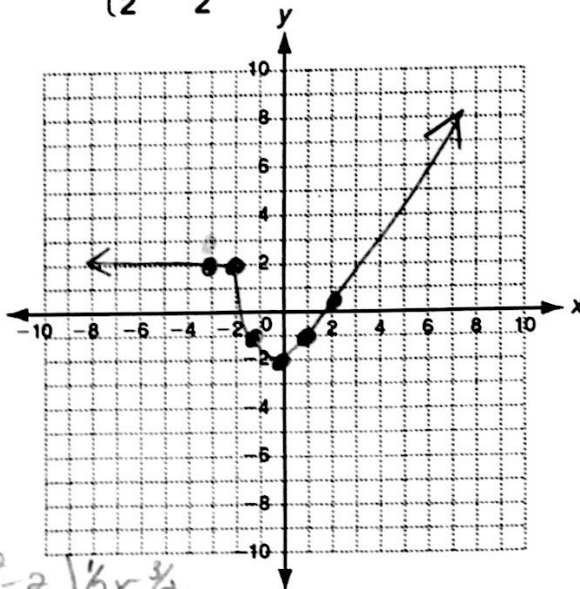
Graph each function.

$$1) f(x) = \begin{cases} x^2 - 2x - 3 & \text{if } x \leq 4 \\ 3x - 7 & \text{if } x > 4 \end{cases}$$



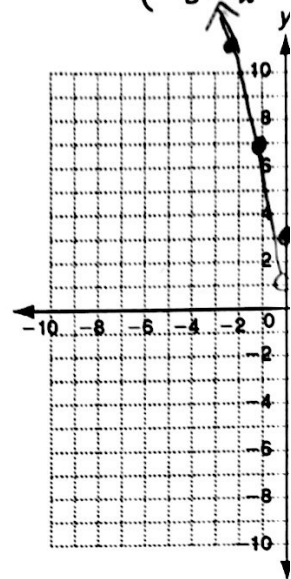
x	$x^2 - 2x - 3$	$3x - 7$
2	-3	X
3	0	X
4	5	5 NI
5	X	8
6	X	11

$$3) f(x) = \begin{cases} 2 & \text{if } -\infty < x \leq -2 \\ x^2 - 2 & \text{if } -2 < x \leq 1 \\ \frac{1}{2}x - \frac{3}{2} & \text{if } 1 < x < \infty \end{cases}$$

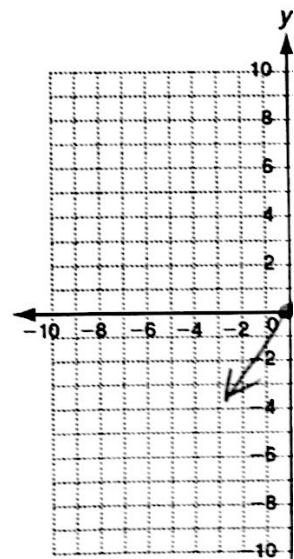


x	2	$x^2 - 2$	$\frac{1}{2}x - \frac{3}{2}$
-3	2	X	X
-2	2	2 NI	X
-1	X	-1	X
0	X	-2	X
1	X	-1	-1 NI
2	X	X	1/2

$$2) f(x) = \begin{cases} 1 - 5x & \\ 3 - x^3 & \\ 5 - x^2 & \end{cases}$$

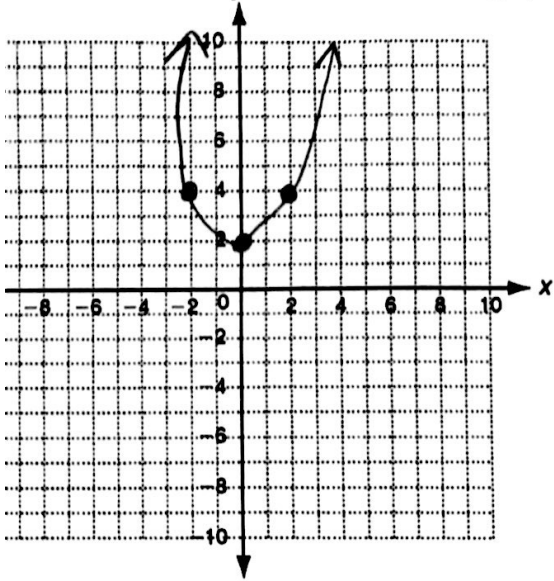


$$4) f(x) = -\frac{5}{3}|x - 3| + 5$$



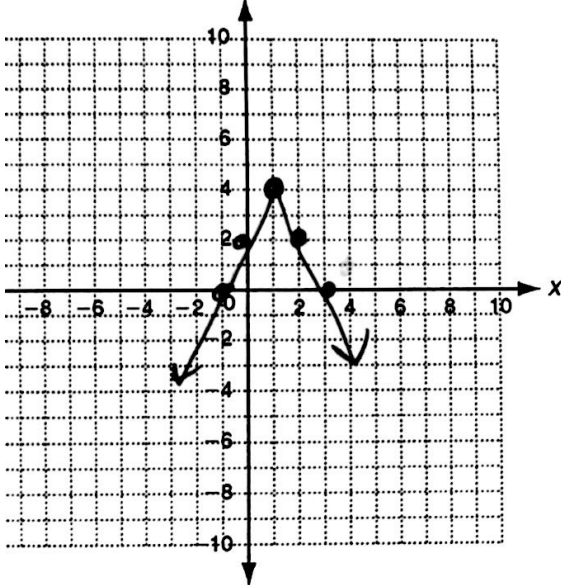
Vertex: (3, 5)

Given $f(x) = 2x^2 + 1$ and $g(x) = f\left(\frac{1}{2}x\right) + 1$, graph $g(x)$.



$$\begin{aligned} f(x) &= 2x^2 + 1 \\ g(x) &= f\left(\frac{1}{2}x\right) + 1 \\ g(x) &= \left(2\left(\frac{1}{2}x\right)^2 + 1\right) + 1 \\ g(x) &= \left(2\left(\frac{1}{4}x^2\right) + 1\right) + 1 \\ g(x) &= \frac{1}{2}x^2 + 2 \end{aligned}$$

Given $f(x) = |x - 2| - 2$ and $g(x) = -2f(x + 1)$, graph $g(x)$.



$$\begin{aligned} g(x) &= -2(f(x+1)) \\ g(x) &= -2(|x+1| - 2 - 2) \\ g(x) &= -2(|x-1| - 2) \\ g(x) &= -2|x-1| + 4 \\ \text{vertex: } &(1, 4) \end{aligned}$$

Given $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 1 \\ 4x - 1 & \text{if } x \geq 1 \end{cases}$, write the rule for each function.

a) $g(x)$, a vertical compression of $f(x)$ by a factor of $\frac{1}{4}$

$$g(x) = \frac{1}{4}f(x) \qquad g(x) = \begin{cases} \frac{1}{4}x^2 + \frac{1}{2} & \text{if } x < 1 \\ x - \frac{1}{4} & \text{if } x \geq 1 \end{cases}$$

$$g(x) = \begin{cases} \frac{1}{4}(x^2 + 2) & \text{if } x < 1 \\ \frac{1}{4}(4x - 1) & \text{if } x \geq 1 \end{cases}$$

b) $h(x)$, a horizontal stretch by a factor of 2

$$h(x) = f\left(\frac{1}{2}x\right) \qquad h(x) = \begin{cases} \frac{1}{4}x^2 + 2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

$$h(x) = \begin{cases} \left(\frac{1}{2}x\right)^2 + 2 & \text{if } x < 1 \\ 4\left(\frac{1}{2}x\right) - 1 & \text{if } x \geq 1 \end{cases}$$

c) $p(x)$, a horizontal translation 3 units left

$$p(x) = f(x+3) \qquad p(x) = \begin{cases} x^2 + 6x + 11 & \text{if } x < -2 \\ 4x + 11 & \text{if } x \geq -2 \end{cases}$$

$$p(x) = \begin{cases} (x+3)^2 + 2 & \text{if } (x+3) < 1 \\ 4(x+3) - 1 & \text{if } (x+3) \geq 1 \end{cases}$$

8) Consider the functions $f(x) = 2x^3 - 3x^2 - 11x + 6$, $g(x) = 3f(\frac{1}{2}x)$ and $h(x) = -g(\frac{1}{2}x)$.

a) Find the x- and y- intercepts of $g(x)$.

$$\begin{aligned} &\underline{f(x)} \\ \text{x-int: } &0 = 2x^3 - 3x^2 - 11x + 6 \\ &\text{x-int} = -3, 3.386, 1 \\ \text{y-int: } &y = 2(0)^3 - 3(0)^2 - 11(0) + 6 \\ &\text{y-int} = 6 \end{aligned}$$

$$\begin{aligned} &\underline{g(x) = 3f(\frac{1}{2}x)} \\ \text{x-int} &= \\ &(-3.386)(2) = \boxed{-6.772} \\ &(1)(2) = \boxed{2} \\ \text{y-int} &= \\ &(6)(3) = \boxed{18} \end{aligned}$$

b) Find the x- and y- intercepts of $h(x)$.

$$\begin{aligned} &\underline{g(x)} \\ \text{x-int} &= \\ &-6.772, 2 \\ \text{y-int} &= 18 \end{aligned}$$

$$\begin{aligned} &\underline{h(x) = -g(\frac{1}{2}x)} \\ \text{x-int} &= \\ &(2)(-6.772) = \boxed{-13.544} \\ &(2)(2) = \boxed{4} \\ \text{y-int} &= \\ &(18)(-1) = \boxed{-18} \end{aligned}$$

Given $f(x) = x^2 - 5x - 14$ and $g(x) = x - 7$, find each function.

9) $(f + g)(x)$

$$\begin{aligned} &f(x) + g(x) = \\ &(x^2 - 5x - 14) + (x - 7) \end{aligned}$$

$$(f + g)(x) = x^2 - 4x - 21$$

10) $(f - g)(x)$

$$\begin{aligned} &f(x) - g(x) = \\ &(x^2 - 5x - 14) - (x - 7) \\ &x^2 - 5x - 14 - x + 7 \end{aligned}$$

$$(f - g)(x) = x^2 - 6x - 7$$

11) $(g - f)(x)$

$$\begin{aligned} &g(x) - f(x) = \\ &(x - 7) - (x^2 - 5x - 14) \\ &x - 7 - x^2 + 5x + 14 \end{aligned}$$

$$(g - f)(x) = -x^2 + 6x + 7$$

12) $(fg)(x)$

$$\begin{aligned} &f(x) \cdot g(x) = \\ &(x^2 - 5x - 14)(x - 7) \\ &x^3 - 7x^2 - 5x^2 + 35x - 14x + 98 \end{aligned}$$

$$(fg)(x) = x^3 - 12x^2 + 21x + 98$$

13) $(\frac{f}{g})(x)$

$$\frac{f(x)}{g(x)}$$

$$\frac{(x^2 - 5x - 14)}{(x - 7)} = \frac{(x + 2)(x - 7)}{(x - 7)} =$$

$$(\frac{f}{g})(x) = x + 2, \text{ where } x \neq 7$$

14) $(\frac{g}{f})(x)$

$$\frac{g(x)}{f(x)}$$

$$\frac{(x - 7)}{(x^2 - 5x - 14)} = \frac{(x - 7)}{(x + 2)(x - 7)} =$$

$$(\frac{g}{f})(x) = \frac{1}{x + 2}, x \neq -2, x \neq 7$$

Let $f(x) = x - 2$ and $g(x) = \frac{8}{x+1}$.

15) Find $f(g(-2))$ and $g(f(-2))$.

$$\begin{aligned} g(-2) &= -8 & f(-2) &= -4 \\ f(-8) &= -10 & g(-4) &= \frac{-8}{3} \\ f(g(-2)) &= -10 & g(f(-2)) &= \frac{-4}{3} \end{aligned}$$

16) Find $f(g(1))$ and $g(f(1))$.

$$\begin{aligned} g(1) &= 4 & f(1) &= -1 \\ f(4) &= 2 & g(-1) &= \text{undef} \\ f(g(1)) &= 2 & g(f(1)) &= \text{undefined} \end{aligned}$$

17) Find $g(f(x))$ and state its domain.

$$g(f(x)) = \frac{8}{(x-2)+1} = \frac{8}{x-1}$$

$$\text{Domain: } \{x \mid x \neq 1\}$$

18) Find $f(g(x))$ and state its domain.

$$f(g(x)) = \left(\frac{8}{x+1}\right) - 2 = \frac{8}{x+1} - 2$$

$$\text{Domain: } \{x \mid x \neq -1\}$$

Find the inverse of each function. Determine whether the inverse is a function, and state its domain and range.

19) $f(x) = 5 - 8x$

$$\begin{aligned} y &= 5 - 8x & f^{-1}(x) &= \frac{x-5}{-8} \\ x &= \frac{5-y}{8} & \text{Domain: } &(-\infty, \infty) \\ x-5 &= -8y & \text{Range: } &(-\infty, \infty) \\ \frac{x-5}{-8} &= y & \text{Function} & \end{aligned}$$

20) $f(x) = \left(\frac{1}{3}x + 2\right)^2$

$$\begin{aligned} y &= \left(\frac{1}{3}x + 2\right)^2 & \sqrt{x-2} &= y \\ x &= \left(\frac{1}{3}y + 2\right)^2 & 3\sqrt{x-6} &= y \\ \sqrt{x} &= \frac{1}{3}y + 2 & f^{-1}(x) &= 3\sqrt{x-6} \\ \sqrt{x-2} &= \frac{1}{3}y & \text{Domain: } &[0, \infty) \\ & & \text{Range: } &[-6, \infty) \end{aligned}$$

21) $f(x) = 3 + \sqrt{x-5}$

$$\begin{aligned} y &= 3 + \sqrt{x-5} & \text{Not a function} & \\ x &= 3 + \sqrt{y-5} & f^{-1}(x) &= (x-3)^2 + 5 \\ x-3 &= \sqrt{y-5} & \text{Domain: } &(-\infty, \infty) \\ (x-3)^2 &= y-5 & \text{Range: } &[5, \infty) \\ (x-3)^2 + 5 &= y & \text{Function} & \end{aligned}$$