Chapter Review Exercises (page 532)

R8.1 (a) We use z critical values to calculate confidence intervals for a proportion. Because $\dfrac{1-0.94}{2}=0.03$, $z^* =$ for a 94% confidence interval can be found by looking for a left-tail area of 0.03. The closest area is 0.0301, corresponding to a critical value of $z^* = 1.88$. *Using technology*: invNorm(area: 0.03, μ : 0, σ : 1) $= -1.881$. Thus, the critical value is $z^* = 1.881$.

(b) We use t critical values to calculate confidence intervals for a mean. Because $n = 58$, df $= 57$. Using Table B and 50 degrees of freedom, $t^* = 2.678$. *Using technology:* invT(area: 0.005, df $= 57$) $= -2.665$, so $t^* = 2.665$.

R8.2 (a) The point estimate $\bar{x}$ is in the exact center of the interval: $\bar{x} = \dfrac{430+470}{2} = 450$ minutes. The margin of error is the distance between the point estimate and the endpoints of the interval: margin of error $= 470 - 450 = 20$ minutes. Because $n = 30$, df $= 29$ and $t^* = 2.045$.

Thus, $20 = 2.045\dfrac{s_X}{\sqrt{30}}$ and the standard error $= \dfrac{s_X}{\sqrt{30}} = \dfrac{20}{2.045} = 9.780$ minutes. Finally, because

$\dfrac{s_X}{\sqrt{30}} = 9.780$, $s_X = 9.780\sqrt{30} = 53.57$ minutes.

(b) This interpretation is incorrect. The confidence interval provided gives an interval estimate for the *mean* lifetime of batteries produced by this company, not individual lifetimes.

(c) No. A confidence interval provides a statement about an unknown population mean, not another sample mean.

(d) If we were to take many samples of 30 batteries and compute 95% confidence intervals for the mean lifetime, about 95% of these intervals will capture the true mean lifetime of the batteries.

R8.3 (a) The parameter p refers to the proportion of all adults aged 18 and older who would say that football is their favorite sport to watch on television. We can't just say that 37% of all adults would say football is their favorite sport because the proportion who choose football will vary from sample to sample.

(b) Random: The sample was random. 10%: The sample size (1000) is less than 10% of all adults. Large Counts: $n\hat{p} = 370 \geq 10$ and $n(1- \hat{p}) = 630 \geq 10$.

(c) $0.37 \pm 1.96\sqrt{\dfrac{0.37(0.63)}{1000}} = (0.3401, 0.3999)$.

(d) We are 95% confident that the interval from 0.3401 to 0.3999 captures the true proportion of all adults who would say that football is their favorite sport to watch on television.

R8.4 (a) The parameter μ refers to the mean IQ score for the 1000 students in the school.

(b) Random: the data are from an SRS. 10%: the sample size (60) is less than 10% of the 1000 students at the school. Normal/Large Sample: $n = 60 \geq 30$.

(c) Using Table B and df $= 50$, the confidence interval is:

$114.98 \pm 1.676\left(\dfrac{14.8}{\sqrt{60}}\right) = 114.98 \pm 3.202 = (111.778, 118.182)$. *Using technology*: (111.79, 118.17) with df $= 59$.

(d) We are 90% confident that the interval from 111.79 to 118.17 captures the true mean IQ score for the 1000 students in the school.

R8.5 Using the conservative guess $\hat{p} = 0.5$, we want $2.576\sqrt{\dfrac{0.5(0.5)}{n}} \leq 0.01$, so

$n \geq \left(\dfrac{2.576}{0.01}\right)^2 (0.5)(0.5) = 16589.44.$ Take an SRS of at least $n = 16{,}590$ adults.

R8.6 (a) *State*: We want to estimate p = the true proportion of all drivers who have run at least one red light in the last 10 intersections they have entered at a 95% confidence level. *Plan*: We should use a one-sample z interval for p if the conditions are satisfied. Random: the drivers were selected at random. 10%: The sample size (880) is less than 10% of all drivers. Large Counts: $n\hat{p} = 171 \geq 10$ and $n(1-\hat{p}) = 709 \geq 10$. *Do*: The confidence interval is

$0.194 \pm 1.96\sqrt{\dfrac{0.194(0.806)}{880}} = (0.168, 0.220).$ *Conclude*: We are 95% confident that the interval from 0.168 to 0.220 captures the true proportion of all drivers who have run at least one red light in the last 10 intersections they have entered.
(b) It is likely that more than 171 respondents have run red lights. We would not expect very many people to claim they *have* run red lights when they have not, but some people will deny running red lights when they have. The margin of error does not account for these sources of bias, only sampling variability.

R8.7 (a) *State*: We want to estimate μ = the true mean measurement of the critical dimension for the engine crankshafts produced in one day at the 95% confidence level. *Plan*: We should construct a one-sample t interval for μ if the conditions are met. Random: The data come from an SRS. 10%: the sample size (16) is less than 10% of all crankshafts produced in one day. Normal/Large Sample: The histogram shows no strong skewness or outliers.
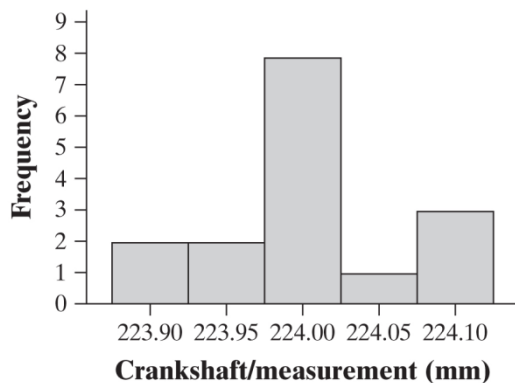
Do: We compute from the measurements that $\bar{x} = 224.002$, $s_x = 0.0618$, and $n = 16$. Thus, df = 15 and $t^* = 2.131$. The confidence interval is

$224.002 \pm 2.131\left(\dfrac{0.0618}{\sqrt{16}}\right) = 224.002 \pm 0.033 = (223.969, 224.035).$ *Conclude*: We are 95% confident that the interval from 223.969 to 224.035 mm captures the true mean measurement of the critical dimension for engine crankshafts produced on this day.
(b) Because 224 is in this interval, it is a plausible value for the true mean. We don't have convincing evidence that the process mean has drifted.

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R8.8 We need $1.96\left(\dfrac{3000}{\sqrt{n}}\right) \le 1000,$ so $n \ge \left(\dfrac{1.96(3000)}{1000}\right)^{2} = 34.57$. So, take a sample of at least 35 pieces of Douglas fir.

R8.9 (a) If we increase the confidence level, then the margin of error must get larger to increase the capture rate of the intervals.
(b) If we quadruple the sample size, the margin of error will decrease by a factor of 2.

R8.10 (a) When we use the sample standard deviation s_{x} to estimate the population standard deviation σ.
(b) The t distributions are wider than the standard Normal distribution and they have a slightly different shape with more area in the tails.
(c) As the degrees of freedom increase, the spread and shape of the t distributions become more like the standard Normal distribution.