Revisiting Extraneous Solutions

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What are we even doing?

~ Determining when a solution is extraneous to a radical equation.

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Not Our First Rodeo

We've looked at extraneous solutions of radical equations before.

But the picture was unclear for many

Suppose we have:

$$(x)^2 = \left((x+12)^{1/2}\right)^2$$

$$x^2 = x+12$$

$$-x-12 \quad -x-12$$

$$x^2-x-12=0$$

$$(x-4)(x+3)=0$$

$$x=4 \quad x=-3$$

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But What's the Problem?

With the solutions of x = 4 and x = -3. But x=-3 is extraneous. Why?

$$x=(x+12)^{1/2}$$

Substitution?

$$4=(4+12)^{1/2} \qquad -3=(-3+12)^{1/2}$$

$$4=4 \qquad -3=(9)^{1/2}$$

$$\boxed{-3=\sqrt{9}}$$

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Let's Rewrite the Expression

Write the rational exponent in radical form.

$$x=(x+12)^{1/2}$$

$$x=\sqrt{x+12}$$

Do we remember what the graph of $f(x)=\sqrt{x}$ looks like?

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How do we know?

So how do we know when a solution is extraneous?

We have: $x=(x+12)^{(1/2)}$

Let's graph BOTH sides of our equation.
So,

   Y1= x

   Y2= $(x+12)^{(1/2)}$

A *solution* will have to satisfy BOTH of these expressions. Where do the functions intersect?

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Let's Try These

$$\left(\sqrt{2x+14}\right)^2 = (x+3)^2$$

$$2x+14 = (x+3)^2$$
$$2x+14 = x^2+6x+9$$
$$0 = x^2+4x-5$$
$$0 = (x-1)(x+5)$$
$$x=1 \quad x=-5$$

$$\left((2x+15)^{(1/2)}\right)^2 = (x)^2$$

$$2x+15 = x^2$$
$$0 = x^2-2x-15$$
$$0 = (x-5)(x+3)$$
$$x=5 \quad \boxed{x=-3}$$
$$\text{ext.}$$

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The Old-Fashioned Way

Of course, there is the classic way of substitution.

BE CAREFUL! Remember that we do not consider both solutions with an even index!

$$(x+3)^2 = \left(\sqrt{x+5}\right)^2$$

$$(x+3)^2 = x+5$$
$$x^2+6x+9 = x+5$$
$$x^2+5x+4 = 0$$
$$(x+1)(x+4) = 0$$
$$\boxed{x=-1} \quad x=-4$$
$$\text{extraneous}$$

$$-1+3 = \sqrt{-1+5}$$
$$2 = 2 \checkmark$$

$$-4+3 = \sqrt{-4+5}$$
$$-1 \neq 1$$

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More Practice!

$$(4x+5)^{(1/2)} \cancel{=} x$$

$$\left(\sqrt{-9x+28}\right)^2 = (-x+4)^2$$

$$-9x+28 = (-x+4)^2$$
$$-9x+28 = x^2-8x+16$$
$$\underline{+9x-28 \quad\quad +9x-28}$$
$$0 = x^2+x-12$$
$$0 = (x+4)(x-3)$$
$$x=-4 \quad x=3$$

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Test Extension Time!

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