Random Variables AP Free-Response Problems key

① a) $P(x \leq 4.2) = .0161$

$\mu_x = 4.5 \quad \sigma_x = 0.14$

He has a 1.61% chance of running less than 4.2 minutes. Therefore, this is very unlikely.

b) $\mu_{Time} = 4.9 + 4.7 + 4.5 + 4.8 = 18.9$ min

$\sigma_{Time} = \sqrt{(0.15)^2 + (0.16)^2 + (0.14)^2 + (0.15)^2} = .3003$

c) $P(x \leq 18.4) = .0480$

② Machine A.

Cost = $10,000

Repair = ($50)(12) = $600 per year $\Rightarrow$ ($600)(3) = $1,800 per 3-year period.

Cost after 3 years = $11,800

Machine B

Cost = $10,500

Expected # of repairs per year = $(0)(.5) + (1)(.25) + (.5)(2) + (3)(.1) = .85$

Expected # of repairs for 3-years = $(3)(.85) = 2.55$

Cost of repairs = $(200)(2.55) = $510

Cost after 3 years = $11,010

Using the expected number of repairs for a 3-year period we found that an expected 2.55 repairs, would be needed. This would cost $510. The total cost for machine B is $11,010. But Machine A costs $50 per month. This would increase the price of machine A by $1800 and have a total cost of $11800. So machine B ~~~~~~

AP Statistics   Name: key

Directions: *Work on these sheets.*

Part 1: Multiple Choice. *Circle the letter corresponding to the best answer.*

1. A random variable is

   D

   (a) a hypothetical list of possible outcomes of a random phenomenon.
   (b) any phenomenon in which outcomes are equally likely.
   (c) any number that changes in a predictable way in the long run.
   (d) a variable whose value is a numerical outcome of a random phenomenon.
   (e) None of the above.

Questions 2, 3, and 4 refer to the following information.
X is a random variable that takes the value A between X = 0 and X = 2 and is zero everywhere else.
X defines a <u>uniform</u> density curve.
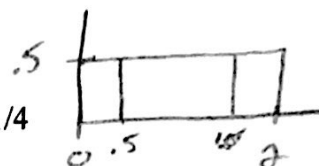
2. The probability that X is between 0.5 and 1.5 is

   B   (a) 1/3.   (b) 1/2.   (c) 3/4.   (d) 1.   (e) 1/4

3. The probability that X is at least 1.5 is

   B   (a) 0.   (b) 1/4.   (c) 1/3.   (d) 1/2.   (e) 1.

4. The probability that X = 1.5 is

   A   (a) 0.   (b) 1/4.   (c) 1/3.   (d) 1/2.   (e) 1.

5. Let the random variable X represent the profit made on a randomly selected day by a certain store. Assume X is Normal with a mean of $360 and standard deviation $50. $P(X > \$400)$ is

   E

   (a) 0.2881.
   (b) 0.8450.
   (c) 0.7881.
   (d) 0.7119
   (e) 0.2119

   $1 - Normalcdf(400, 1000000, 360, 50) = .2119$

6. Let the random variable X represent the amount of money Dan makes doing lawn care in a randomly selected week in the summer. Assume that X is Normal with mean $240 and standard deviation $60. The probability is approximately 0.6 that, in a randomly selected week, Dan will make less than

   C

   (a) $144
   (b) $216
   (c) $255
   (d) $360
   (e) The answer cannot be determined from the information given.

   $invNorm(.6, 240, 60)$

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7. An insurance company has estimated the following cost probabilities for the next year on a particular model of car:

Cost	$0	$500	$1000	$2000
Probability	0.60	0.05	0.13	?

.22

The expected cost to the insurance company is (approximately)

B

(a) $155.
(b) $595.
(c) $875.
(d) $645.
(e) $495.

$E(x) = 595$

8. Suppose we have a *loaded* die that gives the outcomes 1 to 6 according to the probability distribution

X	1	2	3	4	5	6
P(X)	0.1	0.2	0.3	0.2	0.1	0.1

Note that for this die all outcomes are *not* equally likely, as would be if this die were fair. If this die is rolled 6000 times, then $\bar{x}$, the sample mean of the number of spots on the 6000 rolls, should be about

B

(a) 3.
(b) 3.30.
(c) 3.50.
(d) 4.50.
(e) 3.25.

$E(X) = 3.30$

Questions 9 and 10 relate to the following information.
The weight of medium-size oranges selected at random from a bin at the local supermarket is a random variable with mean $\mu = 10$ ounces and standard deviation $\sigma = 1$ ounce.

9. Suppose we pick two oranges at random from the bin. The difference in the weights of the two oranges selected (the weight of the first orange minus the weight of the second orange) is a random variable with a mean (in ounces) of

D

(a) 10.   (b) 1.   (c) 1.41.   (d) 0   (e) 5.

$\mu_{Diff} = \mu_{O_1} - \mu_{O_2}$
$10 - 0 = 0$

10. Suppose we pick two oranges at random from the bin. The difference in the weights of the two oranges selected (the weight of the first orange minus the weight of the second orange) is a random variable with a standard deviation (in ounces) of

E

(a) 0.   (b) 1.   (c) 2.   (d) 2.2.   (e) 1.41.

$\sigma_{Diff} = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{2} =$
1.41

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11. Suppose that a discrete random variable X has the following probability distribution.

X	1	3	5
P(X)	1/4	1/4	1/2

(a) Find the mean μ_X of X.

$$\mu_X = (1)(\tfrac{1}{4}) + (3)(\tfrac{1}{4}) + (5)(\tfrac{1}{2}) = \tfrac{7}{2}$$

(b) Find the variance $(\sigma_X)^2$ of X.

$$\sigma_X^2 = \sum (x - \mu_x)^2 \cdot p(x) = \sqrt{(1.658)} = 1.288$$

(c) Define the new random variable Y = 3X + 1. Use the properties of the mean of linear functions of random variables and your results in the previous parts to find the mean of Y.

$$Y = 3X + 1$$
$$\mu_Y = 3(\tfrac{7}{2}) + 1 = \tfrac{23}{2}$$

(d) Use the properties of the variance of linear functions of random variables to calculate the variance and standard deviation of the new random variable Y.

$$Y = 3X + 1$$
$$\sigma_Y^2 = (3)^2(\sigma_x^2) = 9(1.288)^2 = 14.922$$

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each uniformly distributed between 0 and 1.

(a) The mean of a continuous random variable is the balance point of its density curve. Use this fact to find the mean of Y from the figure.

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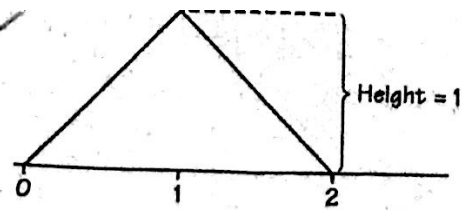
(b) Find the mean of $Y = X_1 + X_2$ using rules for random variables. (Think about the individual density curves for X_1 and X_2.) Verify that the mean of Y is the sum of the mean of X_1 and the mean of X_2.

13. A sample survey contacted an SRS of 663 registered voters in Oregon shortly after an election and asked respondents whether they had voted. Voter records show that 56% of registered voters had actually voted. We will see in Chapter 9 that in this situation the proportion $\hat{p}$ of the sample who voted has approximately the Normal distribution with mean $\mu = 0.56$ and standard deviation $\sigma = 0.019$.

(a) If the respondents answer truthfully, what is $P(0.52 \leq \hat{p} \leq 0.60)$? This is the probability that the statistic $\hat{p}$ estimates the parameter 0.56 within plus or minus 0.04.

$P(\hat{p} \leq .60) - P(\hat{p} \leq .52) = .9647$

(b) In fact, 72% of the respondents said they had voted ($\hat{p} = 0.72$). If respondents answer truthfully, what is $P(\hat{p} \geq 0.72)$? This probability is so small that it is good evidence that some people who did not vote claimed that they did vote.

$P(\hat{p} \geq .72) = 1 - P(\hat{p} < .72) = 1.888 \times 10^{-17} \approx 0$

I pledge that I have neither given nor received aid on this test. _____