Rational and Radical Inequalities

What are we going to do?

~ Solve Rational Inequalities using a graphing calculator and algebraically

~Solve Radical Inequalities using a graphing calculator and algebraically

Technology Works Wonders

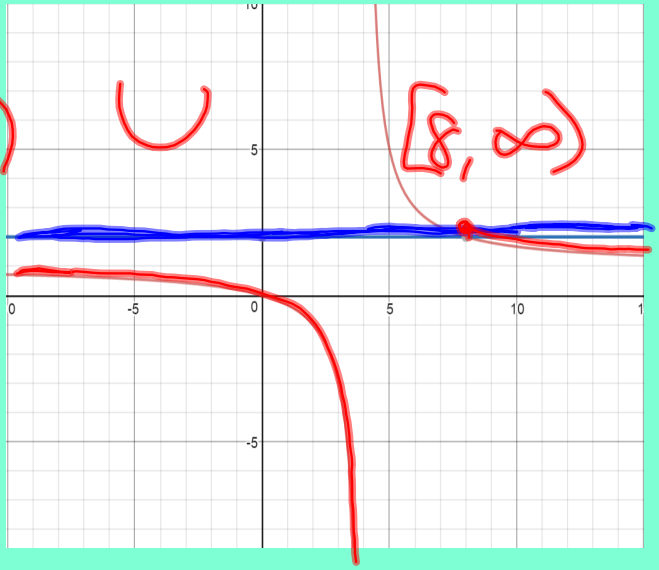
Rational Inequality: an inequality that contains one or more rational expressions

$$\frac{x}{x-4} \le 2$$

Y1: $\frac{x}{x-4}$

Y2: 2

Use your table.

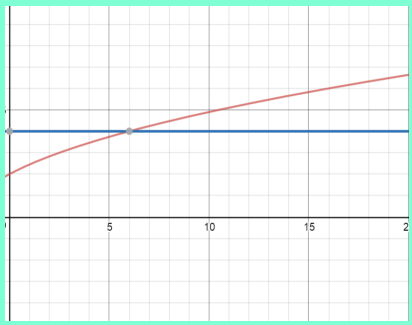


Radical Inequality: an inequality that contains a variable within a radical

$$\sqrt{2x+4} \le 4$$

Y1: $\sqrt{2x+4}$

Y2: 4



Solve the following using your graphing calculator:

a) $\frac{x}{x-3} \geq 4$

b) $\frac{8}{x+1} < -2$

c) $\sqrt{x-3}+2 \leq 5$

d) $\sqrt[3]{x+2} \geq 1$

Solving Rational Inequalities Algebraically

$$\frac{x^2+x-12}{x-1} \leq 0$$

1) Solve for the zeros of the function.

$x^2+x-12=0$

$(x-3)(x+4)=0$

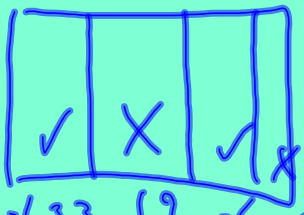
$x=3$ $x=-4$

2) Solve for where the inequality is undefined.

$x-1=0$

$x=1$

-4 1 3



-1.33 12 -6 2.6

≤ 0 ≤ 0 ≤ 0 ≤ 0

3) Create a sign analysis chart.

$(-\infty,-4]\cup[1,3)$

$$\frac{3}{x-2} \leq -1$$

This is the same process as the example above but make it a comparison to 0.

$+1$ $+1$

$\frac{3}{x-2}+1 \leq 0$ LCD $(x-2)$

$\frac{3}{(x-2)}+\frac{1(x+2)}{1(x+2)} \leq 0$

$\frac{3}{(x-2)}+\frac{x-2}{x-2} \leq 0$

① Find zeros

$x+1=0$

$x=-1$

$\frac{x+1}{x-2} \leq 0$

② Undefine

$x-2=0$

$x=2$

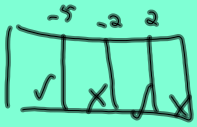
Try These!

$\frac{x+5}{x^2-4} < 0$

$\frac{4}{x+2} > 2$

zeros: $x+5=0$
$x=-5$

undefined: $x^2-4=0$
$x=-2 \quad x=2$



$(-\infty,-5) \cup (-2,2)$

$\frac{-6+5}{(-6)^2-4} < 0$

$\frac{-1}{32} < 0$ ✓

$\frac{-3+5}{(-3)^2-4} = \frac{2}{5} < 0$ X

$\frac{0+5}{(0)^2-4} = \frac{5}{-4} < 0$ ✓

$\frac{5+5}{(5)^2-4} = \frac{10}{21} < 0$ X

$\frac{4}{x+2} > 2$

$-2 \quad -2$

$\frac{4}{(x+2)} - \frac{2}{1} > 0$ LCD: $(x+2)$

$\frac{4}{x+2} - \frac{2(x+2)}{1(x+2)} > 0$

$\frac{4}{x+2} - \frac{2x+4}{x+2} > 0$

$\frac{(-2x)}{(x+2)} > 0$

zeros: $-2x=0$
$x=0$

undef: $x+2=0$
$x=-2$

Solving Radical Inequalities

$\sqrt{x-3}+2 \le 5$

$-2 \quad -2$

$(\sqrt{x-3})^2 \le (3)^2$

$x-3 \le 9$

$x \le 12$

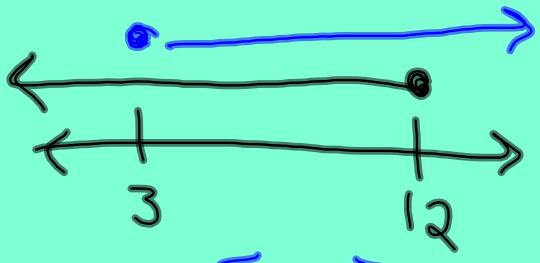
1) Solve for x.

2) Be sure the radicand is not negative.

$x-3 \ge 0$

$x \ge 3$

3) Compare the solution regions.



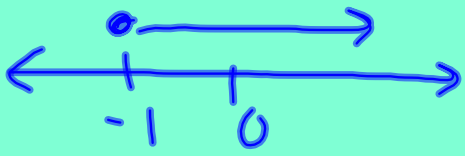
$[3, 12]$

$\sqrt[3]{x+2} \ge 1$

$(\sqrt[3]{x+2})^3 \ge (1)^3$

$x+2 \ge 1$

$x \ge -1$



$[-1, \infty)$

Try these too!

$\left(\sqrt{10x}\right)^2 \le \left(3\sqrt{x+1}\right)^2$

$10x \le 9(x+1)$

$10x \le 9x+9$

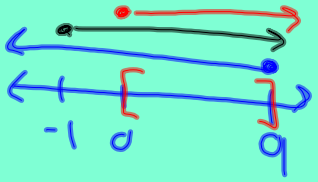
$x \le 9$

$10x \ge 0$

$x \ge 0$

$x+1 \ge 0$

$x \ge -1$



$[0, 9]$

$\sqrt{2h-5} < 1-\sqrt{h-3}$

$\left(\sqrt{2h-5}\right)^2 < \left(1-\sqrt{h-3}\right)^2$

$2h-5 < (1-\sqrt{h-3})(1-\sqrt{h-3})$

$2h-5 < 1-\sqrt{h-3}-\sqrt{h-3}+h-3$

$2h-5 < -2\sqrt{h-3}-2+h$

$-h+2 \quad +2-h$

$(h-3)^2 < \left(-2\sqrt{h-3}\right)^2$

$(h-3)(h-3) < 4(h-3)$

$h^2-6h+9 < 4h-12$

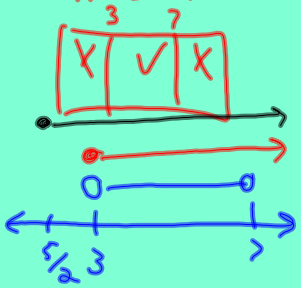
$-4h+12 \quad -4h+12$

$h^2-10h+21 < 0$

$(h-3)(h-7) < 0$

$h=3 \quad h=7$

$3 < h < 7$



$2h-5 \ge 0$

$h \ge 5/2$

$h-3 \ge 0$

$h \ge 3$

$(3, 7)$