Radical Expressions and Rational Exponents

Section 5.6

Totally Radical Objectives:

~Rewrite radical expressions by using rational exponents.

~Simplify and evaluate radical expressions and expressions containing rational exponents.

How Can I write this?

5 and –5 are square roots of 25 because...

2 is the cube root of 8 because...

2 and –2 are fourth roots of 16 because...

So, a is the *n*th root of *b* if ...

Finding Real Roots

The *n*th root of a real number *a* can be written as the radical expression $\sqrt[n]{a}$, where *n* is the **index** of the radical and *a* is the *radicand*.

When a number has more than one root, the radical sign indicates only the principal, or positive, root.

Numbers and Types of Real Roots		
Case	Roots	Example
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ± 2.
Even index; negative radicand	0 real roots	-16 has no real 4th roots.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

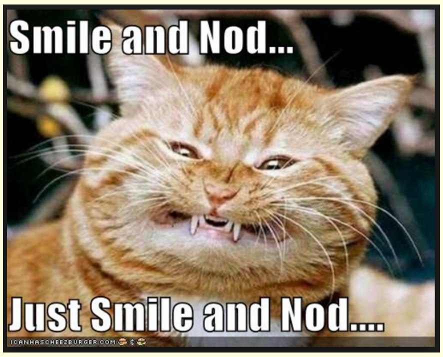
Find all real roots.

A. sixth roots of 64

B. cube roots of –216

C. fourth roots of –1024

Some Review?



Properties of *n*th Roots

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots The *n*th root of a product is equal to the product of the *n*th roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The *n*th root of a quotient is equal to the quotient of the *n*th roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify each expression. Assume that all variables are positive.

A) $\sqrt[4]{81x^{12}}$

B) $\sqrt[4]{16x^4}$

C) $\sqrt[3]{x^7} \cdot \sqrt[3]{x^2}$

Try This!

$\pm 5\sqrt{2}$

a) $\sqrt{50x^3}$

b) $\sqrt[4]{x^8} \bullet \sqrt[3]{x^4}$

c) $\dfrac{\sqrt[3]{x^5}}{4}$

$\sqrt{50} \cdot \sqrt{x^3}$

$\sqrt{50} = \sqrt{2 \cdot 25}$

$\sqrt{50} = \sqrt{2} \cdot \sqrt{25}$

$\sqrt{50} = \pm 5\sqrt{2}$

$\sqrt{50} = \sqrt[2]{2 \cdot 5 \cdot 5}$

$= \pm 5\sqrt{2}$

$50 \to 5, 10$; $10 \to 5, 2$

$\sqrt[2]{x^3} = x^1\sqrt{x^1}$

$\sqrt{x^3} = \sqrt[2]{x \cdot x \cdot x}$

$x\sqrt{x}$

$\sqrt{50} \cdot \sqrt{x^3}$

$\pm 5\sqrt{2} \cdot x\sqrt{x}$

$\pm 5x\sqrt{2x}$

b) $x^2 \cdot x^1\sqrt[3]{x}$

$x^2 \cdot x\sqrt[3]{x}$

$x^3\sqrt[3]{x}$

c) $\dfrac{x\sqrt[3]{x^2}}{4}$

* BONUS *

$$\sqrt[3]{\frac{27x^2y^7}{5}} = \frac{\sqrt[3]{27x^2y^7}}{\sqrt[3]{5}} = \frac{\sqrt[3]{27} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{y^7}}{\sqrt[3]{5}} =$$

$$\frac{3 \cdot \sqrt[3]{x^2} \cdot y^2\sqrt[3]{y}}{\sqrt[3]{5}} = \frac{3y^2\sqrt[3]{x^2y} \cdot \sqrt[3]{5^2}}{\sqrt[3]{5} \cdot \sqrt[3]{5^2}}$$

$$\frac{3y^2\sqrt[3]{x^2y} \cdot \sqrt[3]{25}}{\sqrt[3]{5^3}} = \frac{3y^2\sqrt[3]{25x^2y}}{5}$$

Rational Exponents

ional exponent is an exponent that can be
ssed as $\frac{m}{n}$, where m and n are integers and
. Radical expressions can be written by using
al exponents.

$\frac{m}{n}$ → $\frac{\text{exponent}}{\text{root / index}}$

Rational Exponents

For any natural number n and integer m,

WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the nth root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the nth root raised to the mth power.	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Writing Expressions in Radical Form

Write the expression $(-32)^{\frac{3}{5}}$ in radical form and simplify.

3 → exponent, 5 → root

$$(-32)^{3/5} = \left(\sqrt[5]{-32}\right)^3 = (-2)^3 = -8$$

$$= \sqrt[5]{(-32)^3} = -8$$

Write the expression $64^{\frac{1}{3}}$ in radical form, and simplify.

$$64^{1/3} = \sqrt[3]{64} = 4$$

Try these!

Write the expression $4^{\frac{5}{2}}$ in radical form, and simplify.

$4^{5/2} = (\sqrt{4})^5 = (\pm 2)^5 = \pm 32$

Write the expression $625^{\frac{3}{4}}$ in radical form, and simplify.

$625^{3/4} = (\sqrt[4]{625})^3 = (\pm 5)^3 = \pm 125$

Backwards

Write each expression by using rational exponents.

Dem

A. $\sqrt[8]{13^4}$ num.

$13^{\frac{4}{8}} = 13^{1/2}$

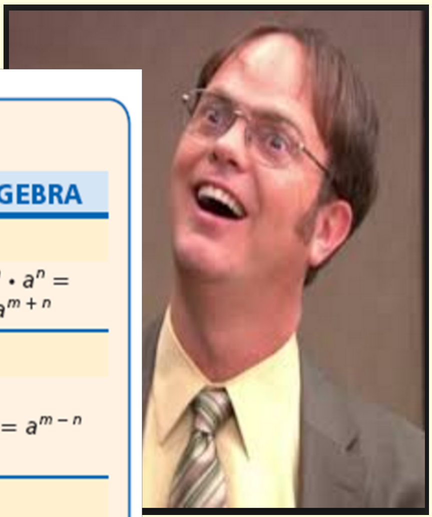
Dem

B. $\sqrt[5]{13^{15}}$ num

15/5

$13^{15/5} = (13)^3$

More Review?



Properties of Rational Exponents

For all nonzero real numbers a and b and rational numbers m and n,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplify each expression.

a) $7^{\frac{7}{9}} \bullet 7^{\frac{11}{9}}$

$7^{18/9} = 7^2 = 49$

b) $\frac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}} = 16^{3/4 - 5/4} =$

$16^{-2/4} = 16^{-1/2} =$

$\frac{1}{16^{1/2}} = \frac{1}{4}$