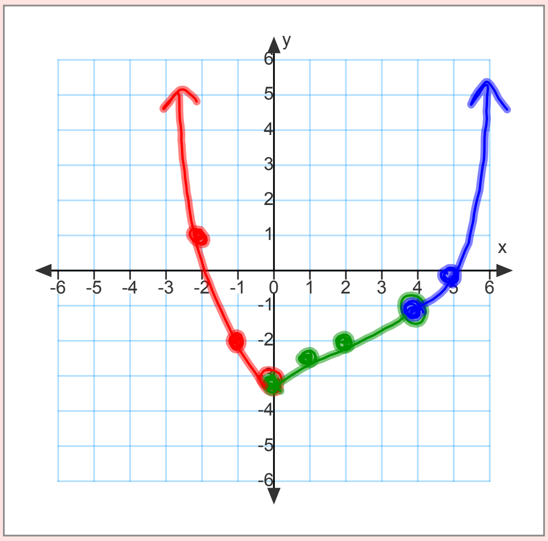
Another Example!

$$g(x) = \begin{cases} x^2 - 3 & \text{if } x < 0 \\ \frac{1}{2}x - 3 & \text{if } 0 \le x < 4 \\ (x-4)^2 - 1 & \text{if } x \ge 4 \end{cases}$$

x	x^2-3	$\frac{1}{2}x-3$	$(x-4)^2-1$
-2	1	X	X
-1	-2	X	X
0	-3 N.I.	-3	X
1	X	-2.5	X
2	X	-2	X
4	X	-1 N.I.	-1
5	X	X	0

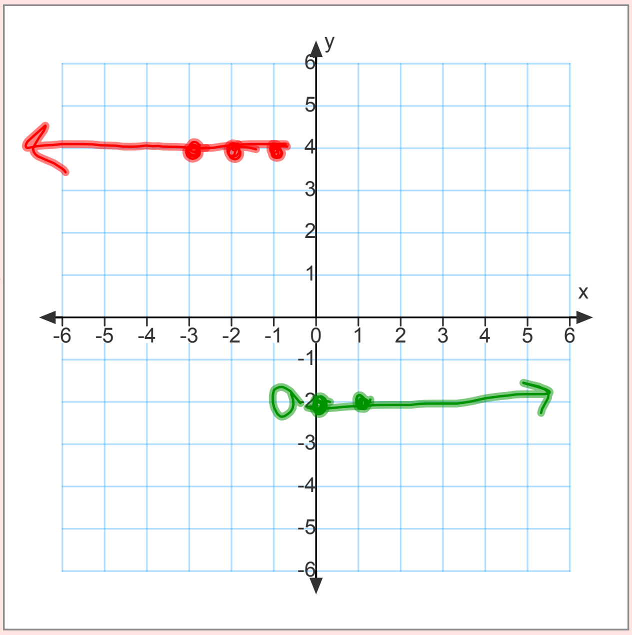


Try These!

a)

$$f(x) = \begin{cases} 4 & \text{if } x \le -1 \\ -2 & \text{if } x > -1 \end{cases}$$

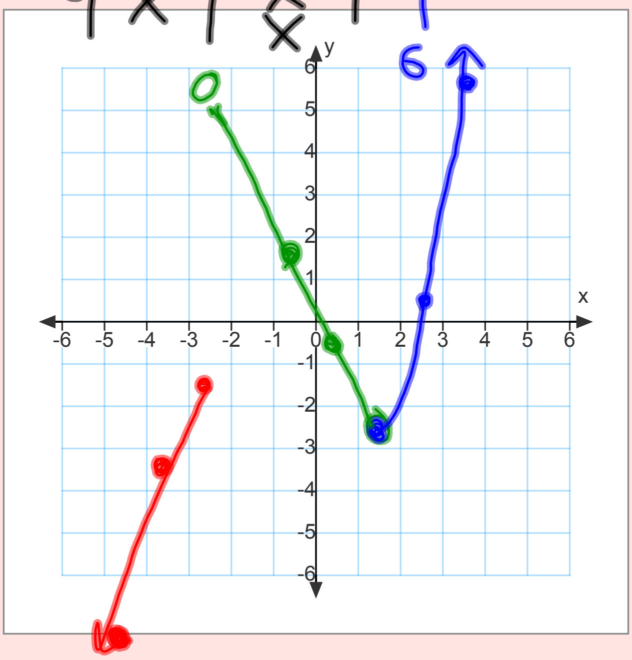
x	4	-2
-3	4	X
-2	4	X
-1	4	-2 N.I.
0	X	-2
1	X	-2



b)

$$g(x) = \begin{cases} 3x+8, & x \le -3 \\ -2x, & -3 < x < 1 \\ x^2 - 3, & x \ge 1 \end{cases}$$

x	3x+8	-2x	x^2-3
-5	-7	X	X
-4	-4	X	X
-3	-1	6 N.I.	X
-1	X	2	X
0	X	0	X
1	X	-2 N.I.	-2
2	X	X	1
3	X	X	6



Just What You've Ordered... Transformations!

Transformations of $f(x)$	
Horizontal Translation	**Vertical Translation**
$f(x) \rightarrow f(x - h)$ left for $h < 0$ right for $h > 0$	$f(x) \rightarrow f(x) + k$ down for $k < 0$ up for $k > 0$
Reflection Across y-axis	**Reflection Across x-axis**
$f(x) \rightarrow f(-x)$ The graph is reflected across the y-axis.	$f(x) \rightarrow -f(x)$ The graph is reflected across the x-axis.
Horizontal Stretch/Compression	**Vertical Stretch/Compression**
$f(x) \rightarrow f\left(\frac{1}{b}x\right)$ stretch for $b > 1$ compression for $0 < b < 1$	$f(x) \rightarrow af(x)$ stretch for $a > 1$ compression for $0 < a < 1$

Caution

Horizontal translations change both the rules and the intervals of piecewise functions. Vertical translations change only the rules.

Example 1)

Given $f(x) = \begin{cases} -\frac{1}{2}x & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } x \geq 0 \end{cases}$ **write the rule** $g(x)$**, a vertical translation up 3.**

$g(x) = f(x) + 3$

$g(x) = \begin{cases} -\frac{1}{2}x + 3, & \text{if } x < 0 \\ \frac{1}{2}x^2 + 3, & \text{if } x \geq 0 \end{cases}$

Example 2)

Given $f(x) = \begin{cases} 2x - 4 & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$ write the rule $g(x)$, a horizontal translation left 2.

+2 +2 * *

$g(x) = f(x+2)$

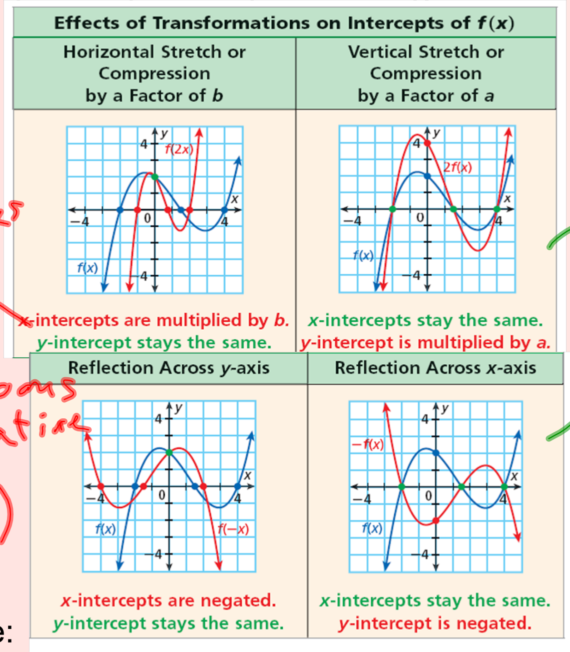
$g(x) = \begin{cases} 2(x+2)-4 & \text{if } x<4 \\ (x+2)^2 & \text{if } x \geq 4 \end{cases}$

$g(x) = \begin{cases} 2x & \text{, if } x<4 \\ x^2+4x+4, & \text{if } x \geq 4 \end{cases}$

Intercepts

When functions are transformed, the x and y intercepts may or may not change.

x-int changes



y-int changes

x becomes negative
$f(-x)$

y becomes negative
$-f(x)$

Example:

Identify the x- and y-intercepts of f(x). Without graphing g(x), identify its x- and y- intercepts.

a) $f(x)=\frac{1}{2}x-3$ and $g(x)=3f(x)$

f(x)-yint:
$y=\frac{1}{2}(0)-3$
y int = -3

$0=\frac{1}{2}x-3$
$+3 \quad +3$
$(2)3=\frac{1}{2}x\,(2)$
6 = x int

$g(x)=3f(x)$
Vertical stretch by 3

x-int: 6
y-int: $=(-3)(3)=-9$

b) $f(x)=x^2-4$ and $g(x)=f(2x)$

y int:
$y=(0)^2-4$
$y=-4$

x-int
$0=x^2-4$
$4=x^2$
$x=2 \quad x=-2$

$g(x)=f(2x)$
Horizontal compression by $\frac{1}{2}$

y-int: -4

$x=2(\frac{1}{2})=1$
$x=-2(\frac{1}{2})=-1$