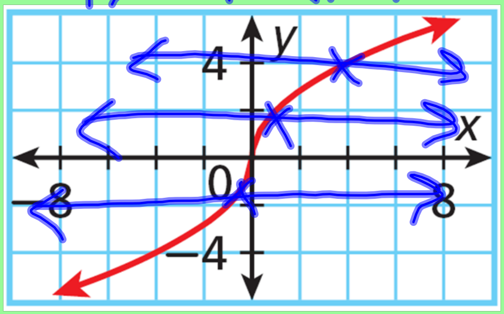
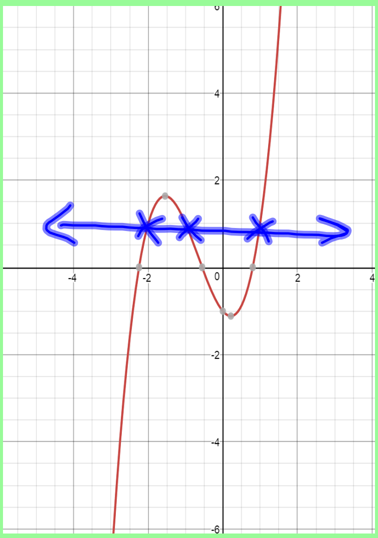
Horizontal Line Test

Use the horizontal-line test to determine whether the inverse of each relation is a function.

a) Inverse function is a function



b) Inverse is NOT a function



How Do We Find the Inverse of a Function?

Simple! Switch x and y. Then, solve!

Find the inverse of $f(x) = \sqrt[3]{x+1}$. Determine whether it is a function, and state its domain and range.

$f(x) = \sqrt[3]{x+1}$

$y = \sqrt[3]{x+1}$

$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$

$x = y+1$

$x^3 - 1 = y$

$f^{-1}(x) = x^3 - 1$

f inverse of x

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Find the inverse of $f(x) = x^3 - 2$. Determine whether it is a function, and state its domain and range.

$f(x) = x^3 - 2$

$y = x^3 - 2$

$x = y^3 - 2$

$+2 \quad +2$

$\sqrt[3]{x+2} = \sqrt[3]{y^3}$

$y = \sqrt[3]{x+2}$

$f^{-1}(x) = \sqrt[3]{x+2}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Try This!

Find the inverse of $f(x)=\frac{5x+9}{6}$ **. Determine whether it is a function, and state its domain and range.**

More on Inverses

You have seen that the inverses of functions are not necessarily functions. When both a relation and its inverses are functions, the relation is called a *one-to-one function*. In a **one-to-one function**, each y-value is paired with exactly one x-value.

$f(x) = x$

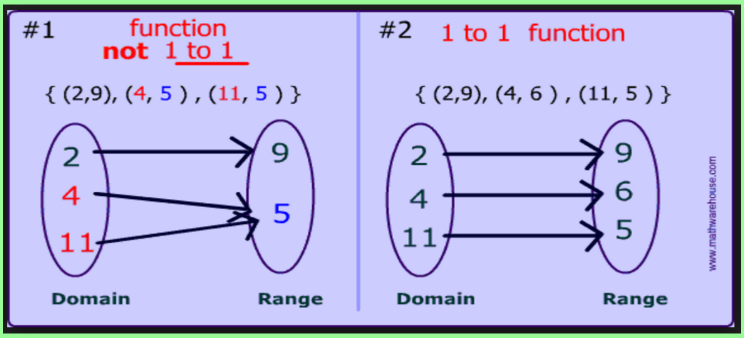
$1 \leftarrow 1$ ✓

$2 \leftarrow 2$

X $f(x) = x^2$

$2 \rightarrow 4$

$-2 \rightarrow 4$



Where it All Leads To

You can use composition of functions to verify that two functions are inverses. Because inverse functions "undo" each other, when you compose two inverses the result is the input value *x*.

Identifying Inverse Functions

WORDS	ALGEBRA	EXAMPLE
If the compositions of two functions equal the input value, the functions are inverses.	If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.	$f(x) = 3x$ and $g(x) = \frac{1}{3}x$ $f(g(x)) = 3\left(\frac{1}{3}x\right) = x$ $g(f(x)) = \frac{1}{3}(3x) = x$

$f(g(x))=x$
AND
$g(f(x))=x$

Determine by composition whether each pair of functions are inverses.

$f(x) = 3x - 1$ and $g(x) = \frac{1}{3}x + 1$

$f(g(x))=$
$x+2$

$g(f(x))=$
$x+2/3$

You Try!

a) **For $x \neq 1$ or 0, $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x} + 1$.**

$f(g(x)) = \frac{1}{g(x)-1} = \frac{1}{\frac{1}{x}+\cancel{1-1}} = \frac{1}{\frac{1}{x}} = \frac{1}{1} \div \frac{1}{x} =$

$1 \cdot \frac{x}{1} = x$

$g(f(x)) = \frac{1}{f(x)} + 1 = \frac{1}{\frac{1}{x-1}} + 1 = x\cancel{-1+1} = x$

$\frac{1}{1} \div \frac{1}{x-1} = 1 \cdot \frac{x-1}{1} = x-1$

b) **$f(x) = \frac{2}{3}x + 6$ and $g(x) = \frac{3}{2}x - 9$**