Name Key

Honors Algebra 2

Date

Midterm Review

Chapter 1 – Function Transformations

1) The revenue from an amusement park ride is given by the admission price of $3 times the number of riders. As part of a promotion, the first 10 riders are free.

a) What kind of transformation describes the change in the revenue based on the promotion?

Vertical shift 30 units down

b)Write a function rule for this transformation.

$g(x) = f(x) - 30 \Rightarrow g(x) = 3x - 30$

2)Suppose the rule $(x, y) \rightarrow (2x, y-3)$ is used to translate a point. If the coordinates of the translated point are (22, 7), what was the original point?

(11, 10)

In 3 – 5, identify the parent function for *g* from its function rule. Then describe what transformations of the parent function it represents.

3) $g(x) = \sqrt{-(x+3)}$ Parent: square root

Reflect over y-axis; Horizontal Translation right 3

4) $g(x) = (3x)^2 - 2$

Parent = Quadratic;

Horizontal compression by 1/3;

Vertical shift down 2

5) $g(x) = x - \sqrt{2}$

Parent = Linear; Vertical translation down $\sqrt{2}$

In 6-7, graph the data from the table. Identify the parent function and describe the transformation that best approximates the data set. Build the function.

6) {(-2, 8), (-1, 1), (0, 0), (1, -1), (2, -8)}

Cubic; Reflect over x-axis

$f(x) = -x^3$

7) {(5, 4), (7, 0), (9, 4), (11, 16)}

Quadratic; Horizontal shift righ

$f(x) = (x-7)^2$

In 8-9, let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.

8) $f(x) = -3x + 7$; horizontal compression by a factor of $\frac{3}{4}$

$g(x) = -4x + 7$

9) $f(x) = -3x + 7$; vertical compression by a factor of $\frac{3}{4}$

$g(x) = -\frac{9}{4}x + \frac{21}{4}$

10) A football kick is modeled by the function $h(x) = -0.0075x^2 + 0.5x + 5$, where h is the height of the ball in feet and x is the horizontal distance in feet that the ball travels. Find the maximum height of the ball to the nearest foot. $h(x) = -0.0075x^2 + 0.5x + 5$

$x = \frac{-b}{2a} = \frac{-0.5}{2(-.0075)} = 33\frac{1}{3}$ $\quad h(33\frac{1}{3}) =$ about $13\frac{1}{3}$ ft

$\boxed{\text{Max height is about 13 feet}}$

11) The height h in feet of a baseball on Earth after t seconds can be modeled by the function $h(t) = -16(t - 1.5)^2 + 36$, where -16 is a constant in ft/s^2 due to Earth's gravity.

a) The gravity on Mars is only 0.38 times that on Earth. If the same baseball were thrown on Mars, it would reach a maximum height 59 feet higher and 2.5 seconds later than on Earth. Describe the transformations that must be applied to make the function model the height of the baseball on Mars. Vert. compression by a factor of 0.38; horizontal shift to the right 2.5; Vertical shift up 59

b) Write a height function for the baseball thrown on Mars.

$h(t) = -6.08(t-4)^2 + 95$

12) A baseball is thrown with a vertical velocity of 50 ft/s from an initial height of 6 feet. The height h in feet of the baseball can be modeled by $h(t) = -16t^2 + 50t + 6$, where t is the time in seconds since the ball was thrown.

a) Approximately how many seconds does it take the ball to reach its maximum height?

about 1.5625 seconds

b) What is the maximum height that the ball reaches?

about 45 ft.

In 13-16, identify the vertex, axis of symmetry, *y*-intercept, *x*-intercept(s) (when possible), direction of opening, domain and range for each function. Then, graph each function.

13) $f(x) = (x + 5)^2$ Vertex: $(-5,0)$; AOS $x=-5$; y-int: $(0,25)$; Up: D: $(-\infty,\infty)$ R: $[0,\infty)$

14) $g(x) = 4x^2 - 2$ Vertex: $(0,-2)$ AOS: $x=0$ y-int: $(0,-2)$ Up D: $(-\infty,\infty)$ R: $[-2,\infty)$

15) $h(x) = -2x(x - 3)$ Vertex: $(1.5,4.5)$ AOS: $x=1.5$ y-int $=(0,0)$; Down: D $(-\infty,\infty)$ R $\cancel{[4.5,\infty)}$ $(-\infty, 4.5]$

16) $p(x) = \frac{1}{4}x^2 + x + 2$ Vertex: $(-2,1)$ AOS $x=-2$ y-int $(0,2)$ Up D: $(-\infty,\infty)$ R: $[1,\infty)$

In 17-18 , use the description to write each quadratic function in the specified form.

17) Write a quadratic function in vertex form that has a y-intercept of 3 and opens upward.

$f(x) = x^2 + 3$ or $f(x) = (x-1)^2 + 2$

18) The parent function $f(x) = x^2$ is reflected across the x-axis and translated 6 units down to create g. Write g in vertex form.

$g(x) = -x^2 - 6$

In , simplify completely. When necessary, write the result in the form *a* + *bi*.

19) $\dfrac{\sqrt{-3}\cdot\sqrt{-6}}{-3\sqrt{2}} = \dfrac{i\sqrt{3}\cdot i\sqrt{6}}{-3\sqrt{2}} =$

1

20) $(-1 + 2i) + (6 - 9i)$

$5-7i$

21) $(4 + 5i)(2 + i)$

$3+14i$

22) $\left(\frac{1}{2}+\frac{3}{4}i\right)-\left(\frac{1}{4}-\frac{5}{4}i\right)$

$\frac{1}{4}+2i$

23). $\dfrac{5+i}{2-i}$

$\frac{9}{5}+\frac{7}{5}i$

24) $i^{24} + i^{13} - i^{12}$

i

Chapter 2 Part 2 – Solving Quadratic Equations

In 25-32 , find the zeros or roots of each function or equation using any of the following methods.

F: Factoring **S: Solving by Square Roots** **CTS: Completing the Square**

QF: Quadratic Formula

Each method must *be used at least twice*. Indicate which method you used. Remember to show work to support your answer.

25) $f(x)=2x^2+4x-12$ $x=-1\pm\sqrt{7}$

26) $56x = 8x^2 + 98$ $x=\frac{7}{2}$

27) $g(x)=x^2+5x-3$ $x=\dfrac{-5\pm\sqrt{37}}{2}$

28) $g(x) = 2x^2 - 25x + 12$ $x=\frac{1}{2}, 12$

29) $f(x) = x^2 + 14x + 24$ $x=-12,-2$

30) $16x^2 - 7 = 2$ $x=\pm\frac{3}{4}$

31) $h(x)=3x^2-3x+\dfrac{3}{4}$ $x=\frac{1}{2}$

32) $h(x) = -x^2 - 6x - 9$ $x=-3$

Complete the square to write each function in vertex form, and identify its vertex.

33) $f(x) = x^2 - 4x - 17$

$f(x)=(x-2)^2-21$ vertex $(2,-21)$

34) $h(x) = 3x^2 - 24x + 15$

$h(x)=3(x-4)^2-33$ vertex $(4,-33)$

Find the type and number of solutions for each equation.

35) $2x^2 + 7 = -4x$

$b^2-4ac=-40$

2 imaginary

36) $4x^2 + 4 = -8x$

$b^2-4ac=0$

1 real solution

Solve each inequality algebraically.

37) $3x^2 + x + 8 \leq 12$

$x = -4/3, 1$

$[-4/3, 1]$

38) $x^2 - 3x - 8 \geq 2$

$x = 5, 2$

$(-\infty, -2] \cup [5, \infty)$

Solve algebraically.

39) The value of a stock is given by S(t) $= t^2 - 6t + 13$, where t is the number of days after the purchase.

(a) Complete the square and write the function in vertex form.

$S(t) = (t-3)^2 + 4$

(b) What is the value of the stock at t = 0? At what other time will the stock have this same value?

$S(0) = 13$; Also at 6 days

(c) What is the vertex? What does the vertex represent in terms of the stock price?

Vertex: (3, 4) represents the minimum value of the stock

40) Fay threw a basketball from the basketball court toward the hoop. The quadratic equation that models the path of the ball is p(t) $= -16t^2 + 20t + 6$. If the hoop is 10 feet high, how long is the ball in the air before it goes through the hoop?

$10 = -16t^2 + 20t + 6$

$-16t^2 + 20t - 4 = 0$

$t = 1/4, 1$

The ball will go in after 1 second.

41) The distance, d, in car lengths, that a drag racer travels during the course of a race is given by $d = 0.8t^2 - 3.5t$, where t is time in seconds.

(a) How long does it take for a racer to travel at least 100 car lengths?

$0.8t^2 - 3.5t \geq 100$ at least 13.57 seconds

(b)During what time period will the racer be more than 50 car lengths but less than 100 car lengths into the race?

$0.8t^2 - 3.5t > 50$ $t \approx 10.39$ sec.

Between 10.39 sec + 13.57 sec

(c)During what time period will the racer be less than 50 car lengths from the start?

Less than 10.39 sec.

Chapter 3 – Polynomials

42) Ms. Liao runs a small dress company. From 1995 through 2005, the number of dresses she made can be modeled by $N(x) = 0.3x^2 - 1.6x + 14$ and the average cost to make each dress can be modeled by $C(x) = -0.001x^2 - 0.06x + 8.3$, where x is the number of years since 1995. Write a polynomial that can be used to model Ms. Liao's total dressmaking costs, $T(x)$, for those years

$N(x) \cdot C(x) = T(x)$

$= (0.3x^2 - 16.x + 14)(-0.001x^2 - 0.06x + 8.3) =$

$T(x) = -0.0003x^4 - 0.0164x^3 + 2.572x^2 - 14.12x + 116.2$

43) A popcorn producer is designing a new box for the popcorn. The marketing department has designed a box with the width 2 inches less than the length and with the height 5 inches greater than the length. The volume of each box must be 24 cubic inches. What is the length of the box?

x: length of box

$V = x(x-2)(x+5)$

$V = x^3 + 3x^2 - 10x$

$x^3 + 3x^2 - 10x = 24$

$x^3 + 3x^2 - 10x - 24 = 0$

$\boxed{x = 3}$

$\boxed{3 \text{ inches}}$

44) A spool of ribbon has a length of $x^3 + x^2$ inches. Write an expression that represents the number of strips of ribbon with a length of $x - 1$ inches that can be cut from one spool.

$(x^3 + x^2) \div (x-1)$ #strips: $x^2 + 2x + 2 + \frac{2}{(x-1)}$

In 45-46, simplify completely.

45) $(2x + y)(2x - y)$

$4x^2 - y^2$

46) $(60 - 16y^2 + y^4) \div (10 - y^2)$

$6 - y^2$

In 47-48, factor completely.

47) $8y^3 - 4y^2 - 50y + 25$

$(2y+5)(2y-5)(2y-1)$

48) $y^5 + 27y^2$

$y^3(y+3)(y^2-3y+9)$

In 49-50, factor the polynomial given that $f(k) = 0$. Write $f(x)$ in completely factored form.

49) $f(x) = 2x^3 - 3x^2 - 8x - 3$; $k = 3$

$f(x) = (x-3)(2x+1)(x+1)$

50) $f(x) = 3x^3 - 19x^2 - 22x + 56$; $k = 7$

$f(x) = (x-7)(3x-4)(x+2)$

In 51-52, list the possible rational zeros of the function using the rational root theorem.

51) $f(x) = x^3 + 7x - 9$

$\pm 1, \pm 3, \pm 9$

52) $f(x) = -3x^4 - 5x^3 - 3x^2 + 7x + 8$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3} \pm \frac{4}{3} \pm \frac{8}{3}$

In 53-54, find all the zeros of the function.

53) $f(x) = x^3 - 2x - 4$

$\{-2, -1 \pm i\}$

54) $h(x) = 4x^4 + x^3 + 25x^2 + 7x - 21$

$\{-1, \frac{3}{4}, \pm i\sqrt{7}\}$

In 55-56, write the simplest polynomial function with the given roots.

55) $-2, i, \sqrt{3}$

$(x+2)(x+i)(x-i)(x-\sqrt{3})(x+\sqrt{3})$

$f(x) = x^5 + 2x^4 - 2x^3 - 4x^2 - 3x - 6$

56) $1 + \sqrt{3}$

$(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})$

$f(x) = x^2 - 2x - 2$

In 57-58, identify the leading coefficient, degree, and end behavior.

57) $r(x) = -6x^4 + 4x^3 - x^2 + 1$

LC: -6 As $x \to +\infty$; $r(x) \to -\infty$

Degree: 4 As $x \to -\infty$; $r(x) \to -\infty$

58) $q(x) = 12 + 8x - 16x^3 - x^2$

LC: -16 As $x \to \infty$, $q(x) \to -\infty$

Degree: 3 A $x \to -\infty$, $q(x) \to \infty$

Chapter 5 Part 1- Rational Expressions

59) The time, t, required to pick some apples varies directly with the number of bushels of apples to be picked, b, and inversely to the number of people picking apples, p, and $t = 1.5$ h when $b = 4$ and $p = 3$. Find b if $t = 1.8$ h and $p = 5$.

$t = \frac{kb}{p}$ $t = 1.5$ $p = 3$ $b = 4$

$k = 1.125$ $t = \frac{1.125b}{p}$ $\Rightarrow$ $\boxed{b = 8 \text{ bushels}}$

In 60-68, simplify completely. Assume that all expressions are defined.

60) $\frac{x^2 - 4x + 4}{5x^2 - 9x - 2} = \frac{x-2}{5x+1}$

61) $\frac{x}{x^2 - 1} \cdot \frac{x^2 - 5x + 4}{2x^2 + 2x} \div \frac{x^2 - 16}{2x} = \frac{x}{(x+1)^2(x+4)}$

62) $\frac{12x^2y}{5y^2} \div \frac{3x^2}{2xy} = \frac{8x}{5}$

63) $\frac{3x+6}{x^2-9} \div \frac{6x^2+12x}{4x+12} = \frac{2}{x(x-3)}$

64) $\frac{2}{x} + \frac{3x+1}{x^2} - \frac{x-2}{x^3} = \frac{5x^2+2}{x^3}$

65) $\frac{3}{x^2 - 7x + 12} + \frac{5x}{x-4} = \frac{15x^2 - 15x + 3}{(x-4)(x-3)}$

66) $\frac{2p}{p^2 - 5p + 6} - \frac{5}{p-2} = \frac{-3(p-5)}{(p-2)(p-3)}$

68) $\frac{\frac{r+6}{r} - \frac{1}{r+2}}{\frac{r^2+4r+3}{r^2+r}} = \left(\frac{r+6}{r} - \frac{1}{(r+2)}\right) \div \left(\frac{r^2+4r+3}{(r^2+r)}\right) =$

$\frac{r+4}{r+2}$

In 69-70, solve each equation.

69) $\frac{x}{x+2} + x = \frac{5x+8}{x+2} =$

$x = 4, \cancel{-2}$

70) $\frac{2x}{x+3} + \frac{5}{x} = \frac{4x^2 + 12x + 21}{x^2 + 3x}$

$x = -3/2, -2$