Do Now:

Simplify the following:

$\sqrt{121}$ $\quad -\sqrt{169}$

$\sqrt{27}$ $\quad \sqrt{1}$

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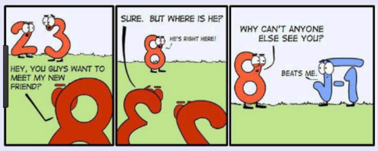
Imaginary Numbers



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Objectives:

Define and use imaginary and complex numbers.

Solve quadratic equations with complex roots.

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Let's look back at the Do Now...

What if we changed those numbers under the radical to be negative?

$\sqrt{-121}$ $\quad -\sqrt{-169}$ $\quad \sqrt{-27}$

$\sqrt{-1}$

Imaginary Numbers

WORDS	NUMBERS	ALGEBRA
An **imaginary number** is the square root of a negative number. Imaginary numbers can be written in the form bi, where b is a real number and i is the imaginary unit. The square of an imaginary number is the original negative number.	$\sqrt{-1} = i$ $\sqrt{-2} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$ $\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$ $\left(\sqrt{-1}\right)^2 = i^2 = -1$	If b is a positive real number, then $\sqrt{-b} = i\sqrt{b}$ and $\sqrt{-b^2} = bi$. $\left(\sqrt{-b}\right)^2 = -b$

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A Little History

Rafael Bombelli

Leonhard Euler

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Try these!

Remember: $\sqrt{-1} = i$

Express the number in terms of *i*.

$\sqrt{-72} = \sqrt{72 \cdot -1}$

$\sqrt{72} \cdot \sqrt{-1}$

$\sqrt{9}\sqrt{8}$

$3\sqrt{8} \cdot i$

$3 \cdot \sqrt{4} \cdot \sqrt{2} \cdot i$

$3 \cdot 2 \cdot \sqrt{2} \cdot i$

$\boxed{\pm 6i\sqrt{2}}$

$\sqrt{-12}$

$\sqrt{12} \cdot \sqrt{-1}$

$\sqrt{4} \cdot \sqrt{3} \cdot i$

$\boxed{\pm 2i\sqrt{3}}$

$2\sqrt{-200}$

$2\sqrt{200} \cdot \sqrt{-1} = i$

$\sqrt{25}\sqrt{8}$

$5 \quad \sqrt{4}\sqrt{2}$

$5 \cdot 2\sqrt{2}$

$2 \cdot 10i\sqrt{2} = \boxed{\pm 20i\sqrt{2}}$

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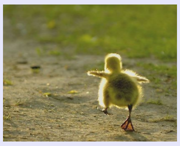
One Step Further



Solving Equations:

$\sqrt{x^2} = \sqrt{-144}$

$x = \sqrt{-144}$

$\boxed{x = \pm 12i}$

$5x^2 + 90 = 0$

$\frac{5x^2}{5} = \frac{-90}{5}$

$\sqrt{x^2} = \sqrt{18}$

$\boxed{x = \pm 3i\sqrt{2}}$

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You Try!

$x^2 + 48 = 0$

-48

$\sqrt{x^2} = -48$

$x = \sqrt{-48}$

$x = \pm 4i\sqrt{3}$

$9x^2 + 25 = 0$

$-25 \quad -25$

$\frac{9x^2}{9} = \frac{-25}{9}$

$\sqrt{x^2} = \frac{-25}{9}$

$x = \pm \frac{5}{3}i$

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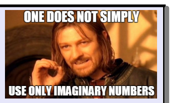
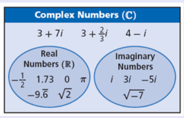
Now It Gets Fun





A **complex number** is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers C.

Every complex number has a **real part** a and an **imaginary part** b.

Real part: a + Imaginary part: bi

$3 + 2i \quad 4 - i$

Real numbers are complex numbers where $b = 0$. Imaginary numbers are complex numbers where $a = 0$ and $b \neq 0$. These are sometimes called *pure imaginary numbers*.

$3 = 3 + 0i \qquad 2i = 0 + 2i$

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

$3 + 2i \neq 3 - 2i$

$3 + 2i = 3 + 2i$

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Examples!

Find the values of x and y that make the equation $4x + 10i = 2 - (4y)i$ true .

Real parts

$4x + 10i = 2 - (4y)i$

Imaginary parts

$\boxed{2x} + \boxed{6i} = \boxed{-8} + \boxed{(20y)i}$

$2x = -8$

$\boxed{x = -4}$

$\frac{-6i}{20i} = \frac{20yi}{20i}$

$-\frac{3}{10} = y$

$\boxed{-8y} + 14i = (7x)i \boxed{-2}$

$\frac{-8y}{-8} = \frac{-2}{-8}$

$y = \frac{1}{4}$

$\frac{14i}{7i} = \frac{(7x)i}{7i}$

$2 = x$

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More Complex Fun!

The complex numbers $-5 + i\sqrt{10}$ and $-5 - i\sqrt{10}$ are related. These solutions are a *complex conjugate* pair. Their real parts are equal and their imaginary parts are opposites. The **complex conjugate** of any complex number $a + bi$ is the complex number $a - bi$.

Find each complex conjugate.

A. $8 + 5i$

$8 - 5i$

B. $6i \Rightarrow 0 + 6i$

$-6i$

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Try These:

Determine the complex conjugate of the following complex numbers:

a) 8-2i

$8 + 2i$

b) 3/4 +5i

$3/4 - 5i$

c) 4i

$-4i$

d) 1

1

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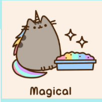
Graphing Complex Numbers

Easier than it seems...

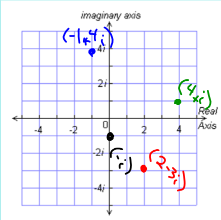


A. $2 - 3i$

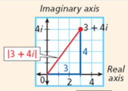
B. $-1 + 4i$

C. $4 + i$

D. $-i$



Absolute Value of a Complex Number

WORDS	ALGEBRA	EXAMPLE
The **absolute value of a complex number** $a + bi$ is the distance from the origin to the point (a, b) in the complex plane, and is denoted $\lvert a + bi\rvert$.	$\lvert a + bi\rvert = \sqrt{a^2 + b^2}$	Imaginary axis  $\lvert 3 + 4i\rvert = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$

$$|a+bi| = \sqrt{a^2+b^2}$$

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Absolute Value

A) $|1 - 2i|$ B) $\left|-\frac{1}{2}\right| = \frac{1}{2}$ C) $|23i| =$ 23

$$|a+bi| = \sqrt{a^2+b^2}$$

$a = 1$

$b = -2$

$$\sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$$

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You Try!

Find each absolute value.

A. $|3 + 5i|$ **B.** $|-13|$ 13 **C.** $|-7i|$ 7

$a = 3$

$b = 5$

$$\sqrt{(3)^2+(5)^2} = \sqrt{34}$$

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Some Operations

$(5 - 2i) - (-2 - 3i)$

$5 - 2i + 2 + 3i$

$7 + i$

$-2i(2 - 4i)$

$-4i + 8i^2$

$-4i + 8(-1)$

$-8 - 4i$

$(3 + 6i)(4 - i)$

$12 - 3i + 24i - 6i^2$

$12 + 21i - 6(-1)$

$12 + 21i + 6$

$18 + 21i$

$(3 + 2i)(3 - 2i)$

$9 - \cancel{6i} + \cancel{6i} - 4i^2$

$9 - 4(-1)$

$9 + 4 = 13$

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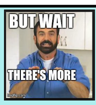
But Wait... There's More



Powers of i		
$i^1 = i$	$i^5 = i^4 \cdot i = 1 \cdot i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$	$i^{10} = -1$
$i^3 = i^2 \cdot i = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$	$i^{11} = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot (-1) = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = 1$

Think remainder when you divide by 4...

Simplify $-6i^{14}$.

$-6,\ \frac{14}{4} = R(2)$

$-6(-1) = 6$

Simplify i^{63}.

$\frac{63}{4} = R3$

$i^{63} = -i$

Simplify i^{43}.

$\frac{43}{4} = R3 = -i$

Divide your power by 4

Write your remainder

$R1 = i$ $R2 = -1$

$R3 = -i$ $R4 = 1$

$R0 =$

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Quick Reminder

We want to simplify but what is wrong?

$$\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

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Same idea,... but with i

Not allowed to have an i in the denominator

$$\frac{(2+8i)(4+2i)}{(4-2i)(4+2i)}$$

$$\frac{8+4i+32i+16i^2}{16+\cancel{8i}-\cancel{8i}-4i^2}$$

(-1)

(-1)

$$\frac{8+36i-16}{16+4}$$

$$\frac{-8+36i}{20}$$

$$\frac{3-i}{2-i}$$

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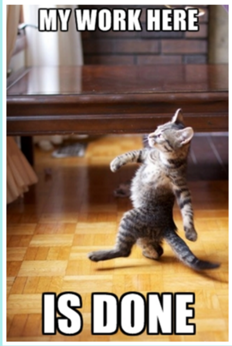

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