

Solve each equation.

$$1. \left(\frac{1}{2}x^2\right) = (-28)^{1/2}$$

$$\sqrt{x^2} = \sqrt{-56}$$

$$x = \pm 2i\sqrt{14}$$

$$2. 3x^2 + 14 = -19$$

$$3x^2 = -33$$

$$x^2 = -11$$

$$x = \pm i\sqrt{11}$$

Find the values of x and y that make each equation true.

$$3. 6x - 2i = (-2y)i + 10$$

$$6x = 10 \quad -2i = (-2y)i$$

$$x = \frac{5}{3} \quad \frac{-2i}{-2i} = \frac{-2y}{-2i}$$

$$1 = y$$

$$4. -40i + 2x = (5y)i - 12$$

$$\frac{-40i}{5i} = \frac{(5y)i}{5i}$$

$$-8 = y \quad \frac{2x}{2} = \frac{-12}{2}$$

$$x = -6$$

$$5. -8y + 14i = (7x)i - 2$$

$$\frac{14i}{7i} = \frac{(7x)i}{7i} \quad \frac{-8y}{-8} = \frac{-2}{-8}$$

$$2 = x \quad y = \frac{1}{4}$$

Find each complex conjugate.

$$9. \sqrt{3}i - 25$$

$$-25 + \sqrt{3}i$$

$$-25 - \sqrt{3}i$$

$$10. -5i + \frac{12}{5}$$

$$\frac{12}{5} - 5i$$

$$\frac{12}{5} + 5i$$

$$11. -2 - 1.5i$$

$$-2 + 1.5i$$

Find each absolute value.

$$12. |-12 + 6i|$$

$$|a + bi| = \sqrt{a^2 + b^2}$$

$$\sqrt{(-12)^2 + (6)^2} = \sqrt{144 + 36} = \sqrt{180}$$

$$6\sqrt{5}$$

$$13. |-7 - 4i|$$

$$\sqrt{(-7)^2 + (-4)^2} = \sqrt{49 + 16} = \sqrt{65}$$

$$14. \left|\frac{1}{2} + \frac{1}{2}i\right|$$

$$\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$$

Add or subtract. Write the result in the form a + bi.

$$15. (8 - i) - (-5 - 4i)$$

$$8 - i + 5 + 4i$$

$$13 + 3i$$

$$16. (2 - 11i) - (10 + 6i)$$

$$2 - 11i - 10 - 6i$$

$$-8 - 17i$$

$$17. \left(\frac{1}{2} + \frac{3}{4}i\right) + \left(-\frac{1}{4} - \frac{5}{4}i\right)$$

$$\frac{1}{2} + \frac{3}{4}i - \frac{1}{4} - \frac{5}{4}i$$

$$\frac{1}{4} - \frac{2}{4}i$$

$$\frac{1}{4} - \frac{1}{2}i$$

Multiply or divide. Write the result in the form $a + bi$.

18. $\frac{-3+7i(1-8i)}{1+8i \cdot (1-8i)}$

$$\frac{-3+24i+7i-56i^2}{1-8i+8i-64i^2}$$

$$\frac{-3+31i-56(-1)}{1-64(-1)} = \boxed{\frac{53+31i}{65}}$$

19. $(-4-9i)(8+2i)$

$$-32-8i-72i-18i^2$$

$$-32-80i-18(-1)$$

$$\boxed{14-80i}$$

20. $\frac{5+i}{2-i}$

$$\frac{10+5i+2i+i^2}{4-2i-2i-i^2}$$

$$\frac{10+7i+(-1)}{4-(-1)} = \boxed{\frac{9+7i}{5}}$$

Simplify.

21. $i^{24} - i^{13} + i^{12}$

$\frac{24}{4} = \text{Remainder } 0 = 1$

$\frac{13}{4} = \text{Remainder } 1 = i$

$\frac{12}{4} = \text{Remainder } 0 = 1$

$$1+i+1 = \boxed{2+i}$$

22. $-4i^{13}$

$\frac{13}{4} = \text{Remainder } 1 = i$

$$-4(i) = \boxed{-4i}$$

23. $6 - 4i^{18}$

$\frac{18}{4} = \text{Remainder } 2 = -1$

$$6 - 4(-1) = 6+4 = \boxed{10}$$

24. In a circuit, the voltage, V , is given by the formula $V = IZ$, where I is the current and Z is the impedance. Both the current and impedance are represented by complex numbers. Find the voltage if the current is $3 + 2i$ and the impedance is $4 - i$.

$$V = IZ$$

$$V = (3+2i)(4-i)$$

$$I = 3+2i$$

$$V = 12 - 3i + 8i - 2i^2$$

$$Z = 4-i$$

$$V = 12 + 5i - 2(-1)$$

$$V = 12$$

$$\boxed{V = 17 + 5i}$$

Challenge Problems

Evaluate and simplify.

25. $\sqrt{-8} \cdot \sqrt{-128}$

$$\pm 2i\sqrt{2} \cdot \pm 8i\sqrt{2}$$

$$\pm 16i \cdot \sqrt{4} =$$

$$\boxed{\pm 32i}$$

27. $(\sqrt{-5})^2$

$$(\pm i\sqrt{5})^2$$

$$i^2(5) = \boxed{-5}$$

26. $\sqrt{-3} \cdot \sqrt{-2} \cdot \sqrt{-6} \cdot \sqrt{-4}$

$$\pm i\sqrt{3} \cdot \pm i\sqrt{2} \cdot \pm i\sqrt{6} \cdot \pm 2i$$

$$\pm 2i^4 \sqrt{36}$$

$$\pm 2(1)(6) = \boxed{\pm 12}$$

28. $\sqrt{-2} \cdot \sqrt{-90} \cdot \sqrt{-5}$

$$\pm i\sqrt{2} \cdot \pm 3i\sqrt{10} \cdot \pm i\sqrt{5}$$

$$\pm 3i^3 \sqrt{100} = \pm 3(-i)(10)$$

$$\boxed{\pm 30i}$$

29. $\sqrt{-3} \cdot \sqrt{12}$

$$\sqrt{-36} = \boxed{\pm 6i}$$

30. $(\sqrt{-2})^5$

$$(\pm i\sqrt{2})^2 \cdot (\pm i\sqrt{2})^2 \cdot \sqrt{-2}$$

$$(-2)(-2)\sqrt{-2}$$

$$\boxed{4\sqrt{-2}}$$