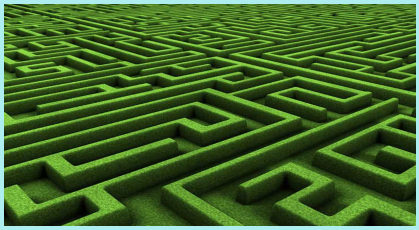
Quadratics

Chapter 2

What questions will we answer?

- How can we transform a quadratic function?
- What are the key features of a quadratic function?

Where to Begin?



- Throughout this entire chapter, we will be analyzing quadratic functions.

~A quadratic function is a function that can be written in the form of

$$f(x) = a(x-h)^2 + k$$

Transformations!

We know the parent function of a quadratic.

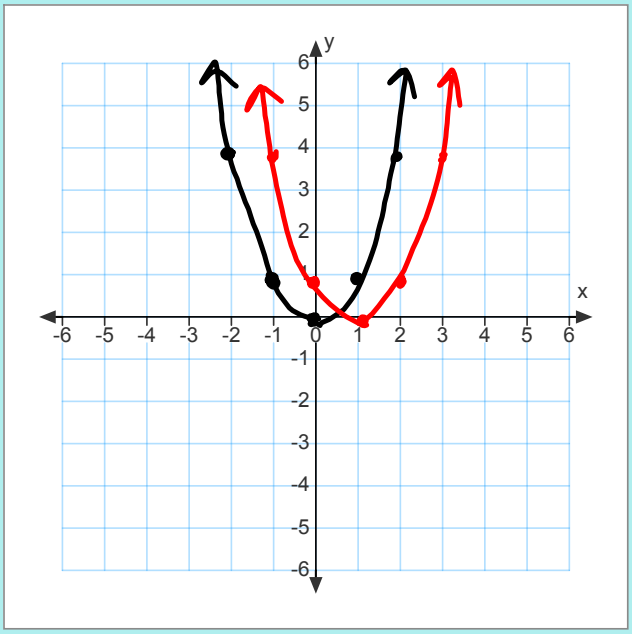
How can we change the parent function for the following? $f(x)=x^2$

What would the transformed graph look like? $f(x)=(x-1)^2$

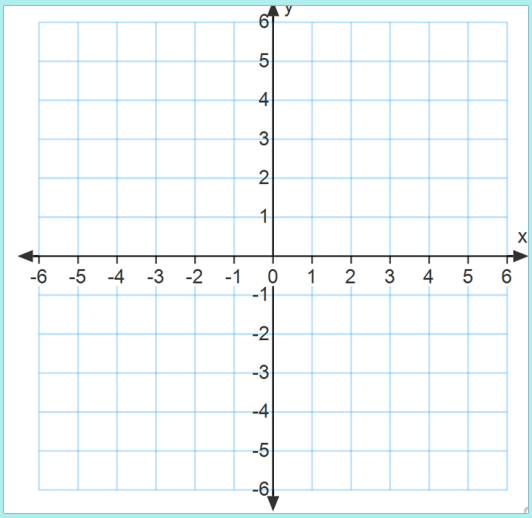
x	y
-2	4
-1	1
0	0
1	1
2	4

→

x	y
-1	4
0	1
1	0
2	1
3	4



- What would the graph of $f(x) = (x-2)^2$ look like?



Each of our quadratic functions form graphs that are ***parabolas***, or U-shaped curves.

x	y
-2	4
-1	1
0	0
1	1
2	4

→

x	y
0	4
1	1
2	0
3	1
4	4

Try this on your own

A) $f(x) = -(x+1)^2 - 5$

x^2

x	y
-2	4
-1	1
0	0
1	1
2	4

→ Horizontal shift left 1

x	y
-3	4
-2	1
-1	0
0	1
1	4

B) Reflect over x-axis

→

x	y
-3	-4
-2	-1
-1	0
0	-1
1	-4

→ Vertical shift down 5

x	y
-3	-9
-2	-6
-1	-5
0	-6
1	-9

$f(x) = 2(x+2)^2 - 3$

x	y
-2	4
-1	1
0	0
1	1
2	4

→

x	y
-4	4
-3	1
-2	0
-1	1
0	4

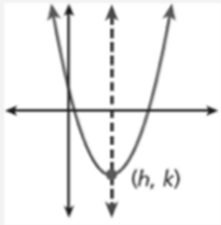
x	y
-4	8
-3	2
-2	0
-1	2
0	8

x	y
-4	5
-3	-1
-2	-3
-1	-1
0	5

Deeper into the Mystery

- Quadratics have key features that describe its characteristics. For each transformation, these may differ.
- One such feature is the ***axis of symmetry***.
- The ***axis of symmetry*** is the line through the vertex of a parabola that divides the parabola into two congruent halves.

Axis of Symmetry Quadratic Functions

WORDS	ALGEBRA	GRAPH
The axis of symmetry is a vertical line through the vertex of the function's graph.	The quadratic function $f(x) = a(x - h)^2 + k$ has the axis of symmetry $x = h$.	

Some Examples:

Identify the axis of symmetry for:

$f(x) = 2(x+2)^2 - 3$

$f(x) = a(x-h)^2 - k$

$x = -2$

$(-2, -3)$

$f(x) = (x-3)^2$

$x = 3$

$(3, 0)$

More Fun!

- Our quadratics functions have been in the same form; Vertex form.

Vertex Form of a Quadratic Function

$$f(x) = a(x-h)^2 + k$$

(h, k)

a indicates a reflection across the *x*-axis and/or a vertical stretch or compression.

h indicates a horizontal translation.

k indicates a vertical translation.

Because the vertex is translated *h* horizontal units and *k* vertical from the origin, the vertex of the parabola is at (*h*, *k*).

Go back to our last two examples and find the vertex of each parabola.

Another Form Exists!

- Standard Form!!
  - The standard form of a quadratic function is where $a \neq 0$

$$f(x) = ax^2 + bx + c$$

Convert the following quadratic function from vertex form to standard form. Then, let's look at the graph.

$$f(x) = 2(x+2)^2 - 3$$

$f(x) = 2[(x+2)(x+2)] - 3$

$f(x) = 2[x^2 + 2x + 2x + 4] - 3$

$f(x) = 2(x^2 + 4x + 4) - 3$

$f(x) = 2x^2 + 8x + 8 - 3$

$f(x) = 2x^2 + 8x + 5$

- What does the value of a tell you about how the parabola will open? What can you expect if a is negative?
- What does the value of c represent?

Properties of a Parabola

Properties of a Parabola

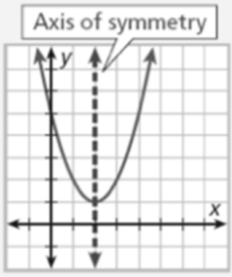
For $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, the parabola has these properties:

The parabola opens upward if $a > 0$ and downward if $a < 0$.

The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

The vertex is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The y-intercept is c.



$x = \frac{-b}{2a}$

Consider the function $f(x) = 2x^2 - 4x + 5$. (a = 2, b = −4, c = 5)

a. Determine whether the graph opens upward or downward.

It opens up.

$x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = 1$

b. Find the axis of symmetry.

The axis of symmetry is $x = 1$.

$2(1)^2 - 4(1) + 5$

$2 - 4 + 5 = 3$

c. Find the vertex.

The vertex is $(1, 3)$.

d. Find the y-intercept.

The y intercept is _____.

You're Up!

$$f(x) = -x^2 - 2x + 3$$

$ax^2 + bx + c$

- Determine whether the graph opens upward or downward.
- Find the axis of symmetry.
- Find the vertex.

Down

a + → up

a - → down

Axis

$\frac{-b}{2a}$ $\frac{-(-2)}{2(-1)} = \frac{2}{-2} = x = -1$

$(-1, 4)$

$f(-1) = -(-1)^2 - 2(-1) + 3$

$-1 + 2 + 3 = 4$

Quadratics

Vertex

$f(x) = a(x-h)^2 + k$

Standard

$f(x) = ax^2 + bx + c$

Up/Down: a

positive → up

negative → down

Axis of Symmetry: $x = h$ || $x = \frac{-b}{2a}$

Vertex: (h, k) || $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

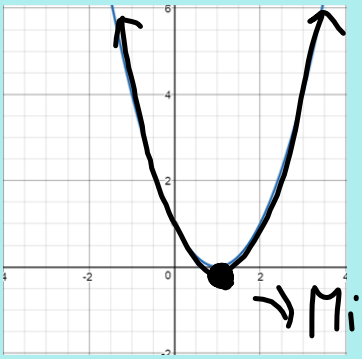
↑ plug x into function

Y-intercept: plug in 0 for x || $(0, c)$

Last Little Bit!

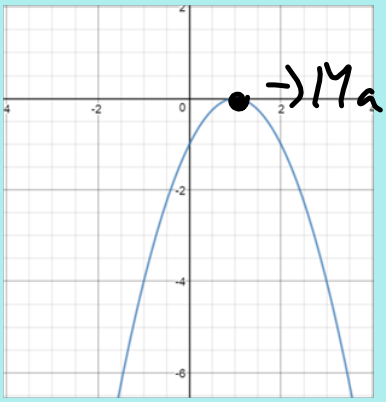
Max and Min

- If a graph opens upward, it will have a ***minimum*** y-value at the y-coordinate of the vertex.



→ Minimum → Vertex

- If the graph opens downward, it will have a ***maximum*** y-value at the y-coordinate of the vertex.



→ Max @ Vertex

Pop Problem!

Write the function below in standard form. ***THEN,*** find the axis of symmetry, vertex, y-intercept, if the parabola open upward or downward and max or min.

$$f(x) = -2(x-1)^2 + 2$$

$f(x) = -2[(x-1)(x-1)] + 2$

$f(x) = -2(x^2 - 2x + 1) + 2$

$f(x) = -2x^2 + 4x - 2 + 2$

$f(x) = -2x^2 + 4x$

Down

Axis of Sym $x = 1$

Vertex $= (1, 2)$

Y-int: $(0, 0)$