

Curve Fitting with Quadratic Models



Objectives

Use quadratic functions to model data.

Use quadratic models to analyze and predict.

Patterns of Differences

How can we determine if the following data is linear?

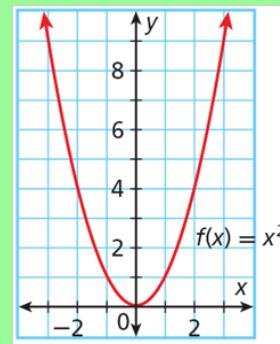
x	5	7	9
y	6	9	12

1st
Difference → $\begin{matrix} \swarrow & \searrow \\ 3 & 3 \end{matrix}$

Similar for Quadratics!

For a set of ordered pairs with equally spaced x -values, a quadratic function has constant nonzero **second** differences, as shown below.

Equally spaced x -values							
x	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
1st differences	-5	-3	-1	1	3	5	
2nd differences	2	2	2	2	2	2	
Constant 2nd differences							



Determine whether the data set could represent a quadratic function. Explain.

x	1	3	5	7	9
y	-1	1	7	17	31

→ Equally spaced x's

1st → $1 - (-1)$ $7 - 1$ $17 - 7$ $31 - 17$

2 → 2 6 10 14

$6 - 2$ $10 - 6$ $14 - 10$

4 4 4

Quadratic !!

Your Turn!

Determine whether the data set could represent a quadratic function. Explain.

A)

x	3	4	5	6	7
y	11	21	35	53	75

↳

$$\begin{matrix} \text{1st} & 10 & 14 & 18 & 22 \\ & \swarrow & \searrow & \swarrow & \searrow \\ & 4 & 4 & 4 & 4 \end{matrix}$$

B)

x	10	9	8	7	6
y	6	8	10	12	14

$$\begin{matrix} \text{1st} & 2 & 2 & 2 & 2 \\ & \downarrow & & & \end{matrix}$$

$$\begin{matrix} & 11 \\ & \swarrow \end{matrix}$$

Back to Lines

How many points do you need to know to find the equation of a line?

$$y = mx + b$$

2 pt



Not the same for quadratics!

$$f(x) = ax^2 + bx + c.$$

3 points

Example:

(1, -5)

Write a quadratic function that fits the points $(1, -5)$,
 $\rightarrow (3, 5)$ and $(4, 16)$.

(x, y)	$f(x) = ax^2 + bx + c$	System in a, b, c
$(1, -5)$	$-5 = a(1)^2 + b(1) + c$	$-5 = a + b + c$
$(3, 5)$	$5 = a(3)^2 + b(3) + c$	$5 = 9a + 3b + c$
$(4, 16)$	$16 = a(4)^2 + b(4) + c$	$16 = 16a + 4b + c$

$$\textcircled{1} \quad -5 = a + b + c$$

$$\textcircled{2} \quad 5 = 9a + 3b + c$$

$$\textcircled{3} \quad 16 = 16a + 4b + c$$

$$\textcircled{1} \quad -5 = a + b + c \quad -5 = a + b + c$$

$$\textcircled{2} \quad (5 = 9a + 3b + c)^{-1}$$

$$\begin{array}{r} -5 = -9a - 3b - c \\ \hline -10 = -8a - 2b \end{array}$$

$$\textcircled{2} \quad 5 = 9a + 3b + c \quad 5 = 9a + 3b + c$$

$$\textcircled{3} \quad (16 = 16a + 4b + c)^{-1}$$

$$\begin{array}{r} -16 = -16a - 4b - c \\ \hline -11 = -7a - b \end{array}$$

$$\begin{array}{r} -10 = -8a - 2b \\ (-11 = -7a - b)^{-2} \\ \hline -10 = -8a - 2b \end{array}$$

$$\begin{array}{r} -10 = -8a - 2b \\ 22 = 14a + 2b \\ \hline 12 = 6a \\ a = 2 \end{array}$$

$$-10 = -8(2) - 2b$$

$$-10 = -16 - 2b$$

$$+16 +16$$

$$6 = -2b \quad \textcircled{b = -3}$$

$$-5 = 2 + (-3) + c \quad f(x) = 2x^2 - 3x - 4$$

$$-5 = -1 + c$$

$$\textcircled{c = -4}$$

Check your Work!

It's very easy to make a mistake!



Substitute or create a table to verify that $(1, -5)$, $(3, 5)$, and $(4, 16)$ satisfy the function rule.

$2(1)^2 - 3(1) - 4$	-5
$2(3)^2 - 3(3) - 4$	5
$2(4)^2 - 3(4) - 4$	16
■	

Ready?

Write a quadratic function that fits the points $(0, -3)$, $(1, 0)$ and $(2, 1)$.



(x, y)	$f(x) = ax^2 + bx + c$	<u>System of Eq</u>
$(0, -3)$	$-3 = a(0)^2 + b(0) + c$	$-3 = c$
$(1, 0)$	$0 = a(1)^2 + b(1) + c$	$0 = a + b + c$
$(2, 1)$	$1 = a(2)^2 + b(2) + c$	$1 = 4a + 2b + c$

$$0 = a + b - 3$$

$$3 = a + b$$

$$1 = 4a + 2b - 3$$

$$4 = 4a + 2b$$

$$(3 = a + b)^{-2}$$

$$4 = 4a + 2b$$

$$-6 = -2a - 2b$$

$$\underline{4 = 4a + 2b}$$

$$-2 = 2a$$

$$a = -1$$

$$3 = -1 + b$$

$$b = 4$$

$$f(x) = -1x^2 + 4x - 3$$

One More Time!

Write a quadratic function that fits the points $(-1, -12)$, $(1, 0)$ and $(2, 9)$.

(x, y)	$f(x) = ax^2 + bx + c$	System
$(-1, -12)$	$-12 = a(-1)^2 + b(-1) + c$	$-12 = a - b + c$
$(1, 0)$	$0 = a(1)^2 + b(1) + c$	$0 = a + b + c$
$(2, 9)$	$9 = a(2)^2 + b(2) + c$	$9 = 4a + 2b + c$

$$\begin{array}{l} -12 = a - b + c \\ 0 = a + b + c \\ 9 = 4a + 2b + c \end{array} \quad \begin{array}{l} -12 = a - b + c \\ 0 = a + b + c \\ \hline -12 = 2a + 2c \end{array}$$

$$\begin{array}{l} (-12 = a - b + c)^2 \\ 9 = 4a + 2b + c \end{array} \quad \begin{array}{l} -24 = 2a - 2b + 2c \\ 9 = 4a + 2b + c \\ \hline -15 = 6a + 3c \end{array}$$

$$\begin{array}{l} (-12 = 2a + 2c)^{-3} \\ -15 = 6a + 3c \end{array} \quad \begin{array}{l} 36 = -18a - 6c \\ -15 = 6a + 3c \\ \hline 21 = -3c \\ \frac{21}{-3} = \frac{-3c}{-3} \\ c = -7 \end{array}$$

$$\begin{array}{l} -12 = 2a - 14 \\ 2 = 2a \\ a = 1 \end{array}$$

$$0 = 1 + b - 7$$

$$0 = -6 + b$$

$$b = 6$$

$$f(x) = x^2 + 6x - 7$$

$$\begin{aligned} & f(x) = (x-2)(x+2) + 7 \\ & f(x) = (x^2 - 4x - 4) + 7 \\ & f(x) = (x-2)^2 + 7 \end{aligned}$$

$$(-2)^2 + 7$$

$$4 + 7$$

$$(0, 11)$$

$$\begin{array}{l} \text{Up} \\ y_{\min.} \end{array}$$

$$9 - 18 - 7 = -16$$

$$y\text{-int} = -7$$

$$(0, -7)$$

$$x = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$$

$$(-3, -16)$$

$$x = 2$$

$$(2, 7)$$

$$(x-h)^2$$

$$(-2, 5), (-1, 0), (1, -2)$$

(x, y)	$f(x) = ax^2 + bx + c$	System
$(-2, 5)$	$5 = a(-2)^2 + b(-2) + c$	$5 = 4a - 2b + c$
$(-1, 0)$	$0 = a(-1)^2 + b(-1) + c$	$0 = a - b + c$
$(1, -2)$	$-2 = a(1)^2 + b(1) + c$	$-2 = a + b + c$

$$\begin{array}{l} 5 = 4a - 2b + c \\ 0 = a - b + c \\ -2 = a + b + c \end{array} \quad \begin{array}{l} 0 = a - b + c \\ -2 = a + b + c \\ \hline -2 = 2a + 2c \end{array}$$

$$\begin{array}{l} 5 = 4a - 2b + c \\ (-2 = a + b + c)^2 \\ -2 = 2a + 2c \end{array} \quad \begin{array}{l} 5 = 4a - 2b + c \\ -4 = 2a + 2b + 2c \\ \hline 1 = 6a + 3c \end{array}$$

$$\begin{array}{l} (-2 = 2a + 2c) \\ 1 = 6a + 3c \end{array} \quad \begin{array}{l} 6 = -6a - 6c \\ 1 = 6a + 3c \end{array}$$

$$\begin{array}{l} 1 = 6a + 3(-\frac{1}{3}) \\ 1 = 6a - 1 \\ 8 = 6a \\ \frac{8}{6} = \frac{4}{3} \\ a = \frac{4}{3} \end{array} \quad \begin{array}{l} 7 = -3c \\ -3 = -3 \\ c = -\frac{1}{3} \end{array}$$

$$\frac{4}{3} - \frac{7}{3} = -\frac{3}{3} = -1$$

$$\begin{array}{l} 0 = a - b + c \\ 0 = \frac{4}{3} - b - \frac{7}{3} \end{array}$$

$$0 = -1 - b \quad f(x) = \frac{4}{3}x^2 - x - \frac{7}{3}$$

$$\begin{array}{l} 1 = -b \\ b = -1 \end{array}$$

(25) ~~$x \mid -1 \mid 0 \mid 1 \mid 2 \mid 3$~~

 ~~$y \mid 0 \mid 1 \mid 0 \mid -3 \mid -8$~~
 $y - (-1) = -2$
 $-3 - (-1) = -2$

1st: $\begin{matrix} 1 & \cancel{-1} & \cancel{-3} & \cancel{-5} \\ -2 & -2 & -2 \end{matrix}$

2nd: $y - (-3) = -2$
 $y = -5$

(26) ~~$x \mid -3 \mid -2 \mid -1 \mid 0 \mid 1$~~

 ~~$y \mid 12 \mid 2 \mid -2 \mid 0 \mid 8$~~
 $8 + 10 = 18$
 $18 / 3 = 6$

1st: $\begin{matrix} 12 & \cancel{2} & \cancel{-2} & \cancel{0} & 8 \\ -10 & -4 & 2 & 0 & 8 \end{matrix}$

2nd: $6 \quad 6 \quad 6$

$y - (-10) = 6$ $? - (2) = -4$
 $y - (-4) = 6$

Quadratic Models

Its tough to be perfect



A **quadratic model** is a quadratic function that represents a real data set. Models are useful for making estimates.

You used a graphing calculator to perform a *linear regression* and make predictions. You can apply a similar statistical method to make a quadratic model for a given data set using **quadratic regression**.

Helpful Hint

The coefficient of determination R^2 shows how well a quadratic function model fits the data. The closer R^2 is to 1, the better the fit. In a model with $R^2 \approx 0.996$, which is very close to 1, the quadratic model is a good fit.

Example

The table shows the cost of circular plastic wading pools based on the pool's diameter. Find a quadratic model for the cost of the pool, given its diameter. Use the model to estimate the cost of the pool with a diameter of 8 ft.

Diameter (ft)	4	5	6	7
Cost	\$19.95	\$20.25	\$25.00	\$34.95

- 1) Enter the data

Stat
EDIT

L1	L2	L3	1
4	19.95		
5	20.25		
6	25.00		
7	34.95		
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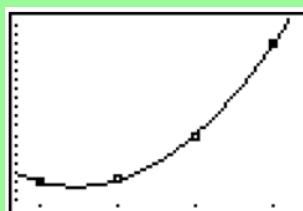
L1(1)=4

- 2) Use QuadReg

```
QuadReg
y=ax^2+bx+c
a=2.4125
b=-21.5625
c=67.6375
R^2=.999808754
```

STAT
CALC
#5 QuadReg

- 3) Graph the function.



- 4) Use the table to find the exact value.

X	Y ₁
4	19.988
5	20.138
6	25.113
7	34.813
8	49.538
9	68.988
10	93.263

X=8

$$\begin{aligned} a &= 2.41 & r^2 &= .9998 \\ b &= -21.56 & c &= 67.64 \end{aligned}$$

$$f(x) = 2.41x^2 - 21.56x + 67.64$$

Diameter: 10 ft

$$f(10) = \$93.44$$

Try this!

The tables shows approximate run times for 16 mm films, given the diameter of the film on the reel. Find a quadratic model for the reel length given the diameter of the film. Use the model to estimate the reel length for an 8-inch-diameter film.

$$a \approx 14.3$$

$$b = -112.4$$

$$c = 430.11$$

$$f(x) = 14.3x^2 - 112.4x + 430.11$$

$$f(8) = 446.38 \text{ ft}$$

Film Run Times (16 mm)		
Diameter (in) x	Reel Length (ft) y	Run Time (min)
5	200	5.55
7	400	11.12
9.25	600	16.67
10.5	800	22.22
12.25	1200	33.33
13.75	1600	44.25