Do Now:

Divide 277 by 12 using long division.

$$\begin{array}{r} 23 \\ 12\overline{)277} \\ -24\downarrow \\ \hline 37 \\ -36 \\ \hline 1 \end{array}$$

$23 + \frac{1}{12}$

Polynomials!

Objectives:

- Identify and graph polynomials
- Add, subtract, multiply and divide polynomials

Some Vocab for the Brain

A **monomial** is a number or a product of numbers and variables with whole number exponents. A

$2, 3x^2, 4h^5z^3t^2\ell^1m^2$

polynomial is a monomial or a sum or difference of monomials. Each monomial in a polynomial is a term. Because a monomial has only one term, it is the simplest type of polynomial.

2

$3x^2+2$

$3x^2+2y^2$

The **degree of a monomial** is the sum of the exponents of the variables.

$3x^2 \rightarrow$ Degree 2

$2a^2b^3 \rightarrow$ Degree 5

$3 \rightarrow$ Degree 0

Examples

Identify the degree of each monomial.

5.6

Degree 0

$8x\cancel{y}^3$

Degree 4

$1+3=4$

a^2bc^3

$2+1+3=6$

More Vocab...

An **degree of a polynomial** is given by the term with the greatest degree. A polynomial with one variable is in standard form when its terms are written in descending order by degree. So, in

Standard Form

Leading coefficient Degree of polynomial

$$5x^3 + 8x^2 + 3x - 17$$

Degree of term:	3	2	1	0

Classifying Polynomials by Degree		
Name	**Degree**	**Example**
Constant	0	-9
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

Examples... again

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

A. $3 - 5x^2 + 4x$

$-5x^2 + 4x + 3$

Quadratic

LC: -5

Degree 2

3 terms

B. $3x^2 - 4 + 8x^4$

$8x^4 + 3x^2 - 4$

LC: 8

Degree 4

3 terms

Quartic

Warming Up!

Perform the indicated operation.

A. $(2x^3 + 9 - x) + (5x^2 + 4 + 7x + x^3)$

$3x^3 + 13 + 6x + 5x^2$

$\boxed{3x^3 + 5x^2 + 6x + 13}$

B. $(3 - 2x^2) - (x^2 + 6 - x)$

$3 - 2x^2 - x^2 - 6 + x$

$-3 - 3x^2 + x$

$\boxed{-3x^2 + x - 3}$

$(5x^3 + 12 + 6x^2) - (15x^2 + 3x - 2)$

$5x^3 + 12 + 6x^2 - 15x^2 - 3x + 2$

$5x^3 - 9x^2 - 3x + 14$

Getting Warmer

Perform the indicated operation.

$4y^2(y^2 + 3)$

$4y^4 + 12y^2$

$(a - 3)(2 - 5a + a^2)$

$2a - \cancel{5a^2} + \cancel{a^3} - 6 + 15a - \cancel{3a^2}$

$\boxed{a^3 - 8a^2 + 17a - 6}$

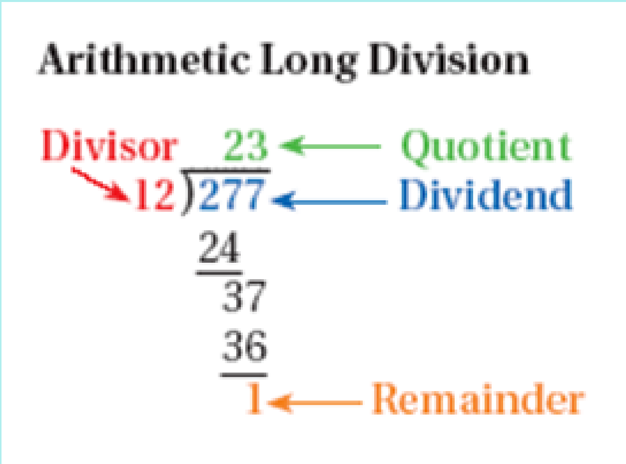
$(y^2 - 7y + 5)(y^2 - y - 3)$

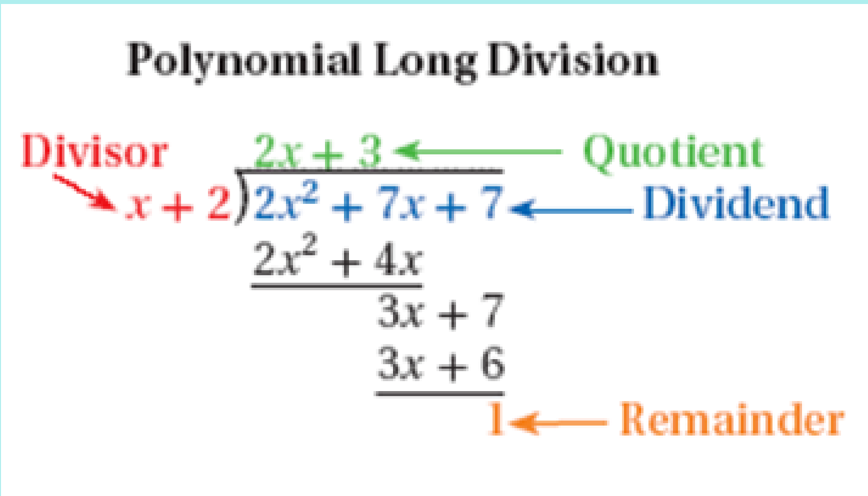
$\cancel{y^4} - \cancel{y^3} - \cancel{3y^2} - \cancel{7y^3} + \cancel{7y^2} + 21y + \cancel{5y^2} - 5y - 15$

$y^4 - 8y^3 + 9y^2 + 16y - 15$

BINGO!

Dividing Polynomials





Example 1)

Divide using long division.
$(-y^2 + 2y^3 + 25) \div (y - 3)$

Step 1 Write the dividend in standard form, including terms with a coefficient of 0.

$(2y^3 - y^2 + 25)$

$2y^3 - y^2 + 0y + 25$

Step 2 Write division in the same way you would when dividing numbers.

Step 3 Divide.

$$\begin{array}{r|l} & 2y^2 + 5y + 15 \\ \hline y-3 & 2y^3 - y^2 + 0y + 25 \\ & -(2y^3 - 6y^2) \\ \hline & 5y^2 + 0y + 25 \\ & -(5y^2 - 15y) \\ \hline & 15y + 25 \\ & -(15y - 45) \\ \hline & 70 \end{array}$$

$2y^2 + 5y + 15 + \frac{70}{y-3}$

Step 4 Write the final answer.

Divide using long division.
$(15x^2 + 8x - 12) \div (3x + 1)$

$$\begin{array}{r|l} & \quad 5x + 1 \\ \hline 3x+1 & 15x^2 + 8x - 12 \\ & -(15x^2 + 5x) \\ \hline & \qquad 3x - 12 \\ & \quad -(3x + 1) \\ \hline & \qquad\quad -13 \end{array}$$

$$\boxed{5x + 1 + \frac{-13}{3x+1}}$$

Another Way

Synthetic division is a shorthand method of dividing a polynomial by a linear binomial by using only the coefficients. For synthetic division to work, the polynomial must be written in standard form, using 0 and a coefficient for any missing terms, and the divisor must be in the form $(x - a)$.

Synthetic Division Method

Divide $(2x^2 + 7x + 9) \div (x + 2)$ by using synthetic division.

WORDS	NUMBERS
Step 1 Write the coefficients of the dividend, 2, 7, and 9. In the upper left corner, write the value of a for the divisor $(x - a)$. So $a = -2$. Copy the first coefficient in the dividend below the horizontal bar.	$\begin{array}{r\|rrr} -2 & 2 & 7 & 9 \\ \hline & 2 & & \end{array}$
Step 2 Multiply the first coefficient by the divisor, and write the product under the next coefficient. Add the numbers in the new column.	$\begin{array}{r\|rrr} -2 & 2 & 7 & 9 \\ & & -4 & \\ \hline & 2 & 3 & \end{array}$
Repeat Step 2 until additions have been completed in all columns. Draw a box around the last sum.	$\begin{array}{r\|rrr} -2 & 2 & 7 & 9 \\ & & -4 & -6 \\ \hline & 2 & 3 & \boxed{3} \end{array}$
Step 3 The quotient is represented by the numbers below the horizontal bar. The boxed number is the remainder. The others are the coefficients of the polynomial quotient, in order of decreasing degree.	$= 2x + 3 + \frac{3}{x + 2}$

Practice!

Divide using synthetic division.

$(3x^4 - x^3 + 5x - 1) \div (x + 2)$

$(3x^4 - x^3 + 0x^2 + 5x - 1) \div (x+2)$

Step 1 Find *a*.

$x+2=0$

$x=-2$

$a=-2$

Step 2 Write the coefficients and *a* in the synthetic division format.

Step 3 Bring down the first coefficient. Then multiply and add for each column.

$$\begin{array}{r|rrrrr} -2 & 3 & -1 & 0 & 5 & -1 \\ & & -6 & 14 & -28 & 46 \\ \hline & 3 & -7 & 14 & -23 & \boxed{45} \\ & x^3 & x^2 & x & c & \text{Remainder} \end{array}$$

$3x^3 - 7x^2 + 14x - 23 + \frac{45}{x+2}$

Step 4 Write the quotient.

Divide using synthetic division.

$(6x^2 - 5x - 6) \div (x + 3)$

$x + 3 = 0$

$a = -3$

$$\begin{array}{r|rrr} -3 & 6 & -5 & -6 \\ & \downarrow & -18 & 69 \\ \hline & 6 & -23 & \boxed{63} \end{array}$$

$6x - 23 + \frac{63}{x+3}$

$$
\begin{array}{r|l}
 & x + 9 \\
x-2 & x^2 + 7x - 5 \\
 & -(x^2 - 2x) \\
 & 9x - 5 \\
 & -(9x - 18) \\
 & 13
\end{array}
$$

$$\boxed{x + 9 + \frac{13}{x-2}}$$