

Factor each and find all zeros.

1) $f(x) = 5x^3 - 11x^2 + 7x - 1$

Possible Rational Roots:

$\pm 1, \pm \frac{1}{5}$

Try $x=1$

$$\begin{array}{r|rrrr} 1 & 5 & -11 & 7 & -1 \\ & \downarrow & & & \\ & 5 & -6 & 1 & 0 \end{array}$$

$5x^2 - 6x + 1 = 0$

$(5x-1)(x-1) = 0$

$x = \frac{1}{5} \quad x = 1$

$x = \frac{1}{5} \quad x = 1$
 ↑
 Mult. of 2

3) $f(x) = 2x^3 + 9x^2 - 2x - 33$

possible rational roots:

$\frac{\pm 1, \pm 3, \pm 11, \pm 33}{\pm 1, \pm 2} = \pm 1, \pm 11, \pm 3, \pm 33, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{11}{2}, \pm \frac{33}{2}$

Try $x = -3$

$$\begin{array}{r|rrrr} -3 & 2 & 9 & -2 & -33 \\ & \downarrow & & & \\ & 2 & 3 & -11 & 0 \end{array} \quad 2x^2 + 3x - 11 = 0$$

$$\frac{-3 \pm \sqrt{(3)^2 - 4(2)(-11)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 88}}{4} =$$

$$\frac{-3 \pm \sqrt{97}}{4}$$

$x = -3 \quad x = \frac{-3 + \sqrt{97}}{4} \quad x = \frac{-3 - \sqrt{97}}{4}$

2) $f(x) = x^4 - 16$

$(x^2 - 4)(x^2 + 4) = 0$

$(x-2)(x+2)(x^2+4) = 0$

$x = 2 \quad x = -2 \quad x = 2i \quad x = -2i$

4) $f(x) = 2x^3 - 3x^2 + 8x - 12$

$0 = (2x^3 - 3x^2) + (8x - 12)$

$0 = x^2(2x-3) + 4(2x-3)$

$0 = (x^2+4)(2x-3)$

$x = 2i \quad x = -2i \quad x = \frac{3}{2}$

$$5) f(x) = 5x^4 + 9x^3 + 3x^2 - x$$

$$0 = 5x^4 + 9x^3 + 3x^2 - x$$

$$0 = x(5x^3 + 9x^2 + 3x - 1)$$

$$x=0 \quad 5x^3 + 9x^2 + 3x - 1 = 0$$

Possible Rational Roots:

$$\pm 1, \pm \frac{1}{5}$$

Try $x = -1$

$$\begin{array}{r|rrrr} -1 & 5 & 9 & 3 & -1 \\ & \downarrow & -5 & -4 & 1 \\ \hline & 5 & 4 & -1 & 0 \end{array}$$

$$5x^2 + 4x - 1 = 0$$

$$(5x-1)(x+1) = 0$$

$$x = \frac{1}{5} \quad x = -1 \quad x = -1 \quad x = 0$$

$$\boxed{x = \frac{1}{5} \quad x = -1 \quad x = 0}$$

Mult. of 2

$$7) f(x) = x^4 - 4x^2 + 3$$

$$0 = x^4 - 4x^2 + 3$$

$$0 = (x^2 - 1)(x^2 - 3)$$

$$0 = (x+1)(x-1)(x^2 - 3)$$

$$\boxed{x = -1 \quad x = 1 \quad x = \sqrt{3} \quad x = -\sqrt{3}}$$

$$6) f(x) = x^4 + 6x^3 + 6x^2 - 4x$$

$$0 = x^4 + 6x^3 + 6x^2 - 4x$$

$$0 = x(x^3 + 6x^2 + 6x - 4)$$

$$x=0 \quad x^3 + 6x^2 + 6x - 4 = 0$$

Possible Rational Roots:

$$\pm 1, \pm 2, \pm 4$$

Try $x = -2$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 6 & -4 \\ & \downarrow & -2 & -8 & 4 \\ \hline & 1 & 4 & -2 & 0 \end{array}$$

$$x^2 + 4x - 2 = 0$$

$$\frac{-4 \pm \sqrt{(4)^2 - 4(1)(-2)}}{2(1)} = \frac{-4 \pm \sqrt{16+8}}{2} =$$

$$\frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm 2\sqrt{6}}{2} = -2 \pm \sqrt{6}$$

$$\boxed{x = 0, x = -2, x = -2 + \sqrt{6} \quad x = -2 - \sqrt{6}}$$

$$8) f(x) = 5x^5 - 15x^4 - 4x^3 + 12x^2 - 9x + 27$$

$$0 = 5x^5 - 15x^4 - 4x^3 + 12x^2 - 9x + 27$$

Possible Rational Roots:

$$\pm 1, \pm 3, \pm 9, \pm 27, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5}, \pm \frac{27}{5}$$

Try $x = 3$

$$\begin{array}{r|rrrrrr} 3 & 5 & -15 & -4 & 12 & -9 & 27 \\ & \downarrow & 15 & 0 & -12 & 0 & -27 \\ \hline & 5 & 0 & -4 & 0 & -9 & 0 \end{array}$$

$$5x^2 - 4x - 9 = 0$$

$$(5x^2 - 9)(x^2 + 1) = 0$$

$$5x^2 - 9 = 0 \quad x^2 + 1 = 0$$

$$5x^2 = 9 \quad x = \pm i$$

$$x^2 = \frac{9}{5}$$

$$x = \pm \sqrt{\frac{9}{5}} = \pm \frac{3}{\sqrt{5}} = \pm \frac{3\sqrt{5}}{5}$$

$$\boxed{x = 3, x = \frac{3\sqrt{5}}{5}, \frac{-3\sqrt{5}}{5}, i, -i}$$