Operations With Rational Expressions

Do Now:

Complete the following operations:

1) Simplify: $\frac{10}{25} = \frac{2 \cdot \cancel{5}}{5 \cdot \cancel{5}} = \frac{2}{5}$

2) $\frac{2}{3} \bullet \frac{8}{9} = \frac{16}{27}$

3) $\frac{8}{10} \div \frac{4}{5}$

$\frac{4}{5} \div \frac{4}{5}$

$\frac{4}{5} \bullet \frac{5}{4} = \frac{20}{20} = 1$

4) $\frac{1}{2} + \frac{2}{5}$

$\frac{(5)1}{(5)2} + \frac{2(2)}{5(2)}$

$\frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

What is a rational expression?

A **rational expression** is a quotient of two polynomials.

$$\frac{x^2-4}{x+2} \qquad \frac{10}{x^2-6} \qquad \frac{x+3}{x-7}$$

$x+2=0$
$x=-2$

$x^2-6=0$ $x=\pm\sqrt{6}$
$+6 \quad +6$
$\sqrt{x^2}=\sqrt{6}$

und.
$x=7$

How do we simplify them?

Just like a fraction!

$$\frac{9}{24}=\frac{3\cdot\cancel{3}}{8\cdot\cancel{3}}=\frac{3}{8}$$

Caution!

When identifying values for which a rational expression is undefined, identify the values of the variable that make the original denominator equal to 0.

Let's Begin!

Simplify. Identify any *x*-values for which the expression is undefined.

1) $\dfrac{10x^8}{6x^4} = \dfrac{10}{6} \cdot \dfrac{x^8}{x^4} = \boxed{\dfrac{5}{3}x^4}$

$6x^4 = 0$

$x = 0$

und.

$\dfrac{x^8}{x^4} = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^4$

2) $\dfrac{x^2 + x - 2}{x^2 + 2x - 3}$

$\dfrac{(x+2)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \boxed{\dfrac{x+2}{x+3}}$

$x+3=0$ $x-1=0$

$x=-3$ $x=1$ ← und.

3) $\dfrac{3x + 4}{3x^2 + x - 4}$

$\dfrac{\cancel{(3x+4)}}{\cancel{(3x+4)}(x-1)} = \dfrac{1}{x-1}$

$3x+4=0$ $x=1$

$x=-\dfrac{4}{3}$

Your Turn!

Simplify. Identify any *x*-values for which the expression is undefined.

1) $\dfrac{16x^{11}}{8x^2} = \dfrac{16}{8} \cdot \dfrac{x^{11}}{x^2} = 2x^9$

$x = 0$ und.

2) $\dfrac{6x^2 + 7x + 2}{6x^2 - 5x - 5} = \dfrac{(3x+2)(2x+1)}{6x^2 - 5x - 5}$

More Simplifying

Simplify $\frac{4x - x^2}{x^2 - 2x - 8}$ **. Identify any x values for which the expression is undefined.**

$$\frac{x(4-x)}{(x-4)(x+2)} = \frac{x(-x+4)}{(x-4)(x+2)} = \frac{(-1)(x)\cancel{(x-4)}}{\cancel{(x-4)}(x+2)}$$

$$\boxed{\frac{-x}{x+2}}$$

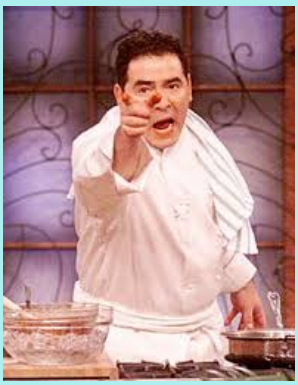
Try this!

Simplify $\frac{10 - 2x}{x - 5}$. Identify any x values for which the expression is undefined.

$$\frac{2(5-x)}{(x-5)} = \frac{2(-1)\cancel{(x-5)}}{\cancel{(x-5)}} = -2$$

Simplify $\frac{-x^2 + 3x}{2x^2 - 7x + 3}$. Identify any x values for which the expression is undefined.

Let's Kick it Up a Knotch!



Multiplying Rational Expressions
1. Factor all numerators and denominators completely.
2. Divide out common factors of the numerators and denominators.
3. Multiply numerators. Then multiply denominators.
4. Be sure the numerator and denominator have no common factors other than 1.

Multiplication

Multiply. Assume that all expressions are defined.

$$\frac{3x^5y^3}{2x^3y^7} \cdot \frac{10x^3y^4}{9x^2y^5} = \frac{30x^8y^7}{18x^5y^{12}}$$

$$\frac{30}{18} \cdot \frac{x^8}{x^5} \cdot \frac{y^7}{y^{12}} = \boxed{\frac{5x^3}{3y^5}}$$

$$\frac{x-3}{4x+20} \cdot \frac{x+5}{x^2-9}$$

$$\frac{1\cancel{(x-3)}}{4\cancel{(x+5)}} \cdot \frac{1\cancel{(x+5)}}{\cancel{(x-3)}(x+3)} = \frac{1}{4(x+3)} = \frac{1}{4x+12}$$

You Try!

Multiply. Assume that all expressions are defined.

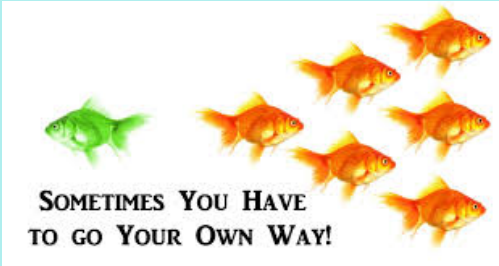
$$\frac{x}{15} \cdot \frac{x^7}{2x} \cdot \frac{20}{x^4}$$

$$\frac{x}{15} \cdot \frac{x^6}{2} \cdot \frac{20}{x^4} = \frac{20x^7}{30x^4} = \frac{2x^3}{3}$$

$$\frac{10x - 40}{x^2 - 6x + 8} \cdot \frac{x + 3}{5x + 15}$$

$$\frac{\cancel{10}^{2}\cancel{(x-4)}}{\cancel{(x-4)}(x-2)} \cdot \frac{\cancel{(x+3)}}{\cancel{5}\cancel{(x+3)}} = \frac{2}{x-2}$$

With a partner, divide the following:



$$\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5} = \frac{5x^4}{8x^2y^2} \cdot \frac{8y^5}{15}$$

$$\frac{40x^4y^5}{120x^2y^2} = \frac{40}{120} \cdot \frac{x^4}{x^2} \cdot \frac{y^5}{y^2} = \frac{1x^2y^3}{3}$$

$$\frac{2x^2 - 7x - 4}{x^2 - 9} \div \frac{4x^2 - 1}{8x^2 - 28x + 12} = \frac{2x^2 - 7x - 4}{x^2 - 9} \cdot \frac{8x^2 - 28x + 12}{4x^2 - 1}$$

$(8x-4)(x-3)$

$$\frac{(x-4)\cancel{(2x+1)}}{(x+3)\cancel{(x-3)}} \cdot \frac{4\cancel{(2x-1)}\cancel{(x-3)}}{\cancel{(2x-1)}\cancel{(2x+1)}} = \frac{4x-16}{x+3}$$

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \div \frac{x^4 + 2x^3 - 8x^2}{x^2 - 16}$$

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \cdot \frac{x^2 - 16}{x^4 + 2x^3 - 8x^2}$$

$$\frac{\cancel{x^2}(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} \cdot \frac{\cancel{(x+4)}(x-4)}{\cancel{(x^2)}\cancel{(x+4)}(x-2)} = \frac{(x+3)(x-4)}{(x-1)(x-2)}$$

$$\frac{2x^2 - 7x - 4}{x^2 - 9} \div \frac{4x^2 - 1}{8x^2 - 28x + 12}$$

$$\frac{4x^3y^2}{2x^6y} \div \frac{xy+2y}{x^2-3x-10}$$

$$\frac{4x^3y^2}{2x^6y} \cdot \frac{x^2-3x-10}{xy+2y}$$

$$\frac{2y}{x^3} \cdot \frac{(x-5)\cancel{(x+2)}}{y\cancel{(x+2)}}$$

$$\frac{2\cancel{y}}{x^3} \cdot \frac{(x-5)}{\cancel{y}} = \frac{2(x-5)}{x^3}$$

Solving

Solve. Check your solution.

$$\frac{x^2 - 25}{x - 5} = 14$$

$$\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} = 14$$

$x+5=14$

$x=9$

$$\frac{4x^2 - 9}{2x + 3} = 5$$

$$\frac{\cancel{(2x+3)}(2x-3)}{\cancel{(2x+3)}} = 5$$

$2x-3=5$

$x=4$

$$\frac{x^2+3x-10}{x-2} = 7$$

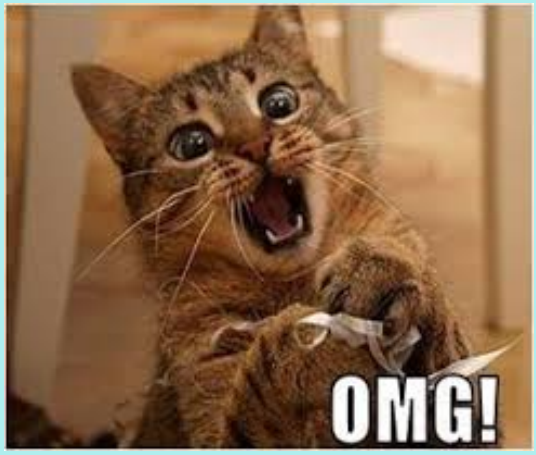
$$\frac{(x+5)\cancel{(x-2)}}{\cancel{(x-2)}} = 7$$

$x+5=7$

$x=2$

No sol.

Adding and Subtracting Rational Expressions



What do we need to add fractions?

Add or subtract. Identify any *x*-values for which the expression is undefined.

$$\frac{(x-3)}{(x+4)} + \frac{(x-2)}{(x+4)} = \frac{x-3+x-2}{x+4} = \frac{2x-5}{x+4}$$

$$\frac{1}{10} + \frac{2}{10} = \frac{3}{10}$$

$$\frac{3x-4}{x^2+1} - \frac{6x+1}{x^2+1} = \frac{-3x-5}{x^2+1}$$

$3x-4-6x-1$

$x^2+1=0$

$\sqrt{x^2}=\sqrt{-1}$

$x=\pm i$

$$\frac{3x^2-5}{3x-1} - \frac{2x^2-3x-2}{3x-1} = \frac{x^2+3x\cdot 3}{3x-1}$$

$3x^2-5-2x^2+3x+2$

But what if we don't have a common denominator?

Least Common Multiple (LCM) of Polynomials
To find the LCM of polynomials:
1. Factor each polynomial completely. Write any repeated factors as powers. For example, $x^3 + 6x^2 + 9x = x(x+3)^2$.
2. List the different factors. If the polynomials have common factors, use the highest power of each common factor.

$$\frac{1}{(x+2)} + \frac{1}{x^2+4x+4}$$

$$\frac{1}{(x+2)} + \frac{1}{(x+2)(x+2)}$$

Example Time!

Find the least common multiple for each pair.

A. $4x^2y^3$ and $6x^4y^5$

$(4x^2y^3)\cdot 3x^2y^2$ $(6x^4y^5)\cdot 2$

$12x^4y^5$ $12x^4y^5$

B. $x^2 - 2x - 3$ and $x^2 - x - 6$

$(x-3)(x+1)(x+2)$ $(x-3)(x+2)(x+1)$

You're Turn

Find the least common multiple for each pair.

a. $4x^3y^7$ **and** $3x^5y^4$

b. $x^2 - 4$ **and** $x^2 + 5x + 6$

Its Time!

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{(x-3)}{x^2+3x-4}+\frac{(2x)}{(x+4)}$$

$$\frac{(x-3)}{(x+4)(x-1)}+\frac{(2x)(x-1)}{(x+4)(x-1)}=\frac{\quad}{(x+4)(x-1)}$$

$$\frac{x-3+2x^2-2x}{(x+4)(x-1)}=\frac{2x^2-x-3}{(x+4)(x-1)}$$

$$\frac{x}{x+2}+\frac{-8}{x^2-4}$$

$$\frac{(x-2)(x)}{(x-2)(x+2)}+\frac{(-8)}{(x+2)(x-2)}=\frac{x^2-2x-8}{(x-2)(x+2)}=\frac{(x-4)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$\frac{x-4}{x-2}$$

$$\frac{3x}{2x-2}+\frac{3x-2}{3x-3}$$

$$9x+6x-4$$

$$\frac{3\cdot 3x}{3\cdot 2(x-1)}+\frac{(3x-2)\cdot 2}{3(x-1)\cdot 2}=\frac{15x-4}{6x-6}$$

Subtract $\frac{3x-2}{2x+5}-\frac{2}{5x-2}$. Identify any *x*-values for which the expression is undefined.

Now, You Try!

Subtract $\frac{2x^2 - 30}{x^2 - 9} - \frac{x + 5}{x + 3}$. Identify any x-values for which the expression is undefined.

$$\frac{x}{x + 3} + \frac{2x + 6}{x^2 + 6x + 9}$$