Do Now:

a) $\sqrt{72}$

$\sqrt{36}\ \sqrt{2}$

$\pm 6\sqrt{2}$

b) $\sqrt{180}$

$\sqrt{36}\ \sqrt{5}$

$\pm 6\sqrt{5}$

Radical Expressions and Rational Exponents

Section 5.6

Totally Radical Objectives:

~Rewrite radical expressions by using rational exponents.

~Simplify and evaluate radical expressions and expressions containing rational exponents.

How Can I write this?

5 and –5 are square roots of 25 because...

$(5)^2 = 25 \quad (-5)^2 = 25$

2 is the cube root of 8 because...

$(2)^3 = 8$

2 and –2 are fourth roots of 16 because...

$(2)^4 = (-2)^4 = 16$

So, a is the *n*th root of *b* if ...

$a^n = b$

$a = \sqrt[n]{b}$

Finding Real Roots

The *n*th root of a real number *a* can be written as the radical expression $\sqrt[n]{a}$, where *n* is the **index** of the radical and *a* is the *radicand*.

$\sqrt[n]{a}$ index

When a number has more than one root, the radical sign indicates only the principal, or positive, root.

Numbers and Types of Real Roots		
Case	Roots	Example
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ±2.
Even index; negative radicand	0 real roots	−16 has no real 4th roots.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

$\sqrt[x]{}$

Find all real roots.

A. sixth roots of 64

$\sqrt[6]{64} = \pm 2$

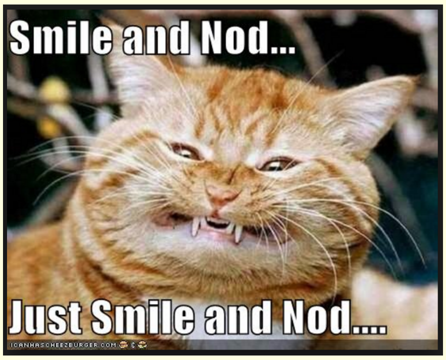
B. cube roots of −216

$\sqrt[3]{-216} = -6$

C. fourth roots of −1024

$\sqrt[4]{-1024} =$ none

Some Review?



Properties of *n*th Roots

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots The *n*th root of a product is equal to the product of the *n*th roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The *n*th root of a quotient is equal to the quotient of the *n*th roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify each expression. Assume that all variables are positive.

A) $\sqrt[4]{81x^{12}}$

$\sqrt[4]{81}$ $\quad$ $\sqrt[4]{x^{12}}$

$\boxed{\pm 3x^3}$

$\sqrt[4]{x^{12}}$

$\sqrt[4]{x^4} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^4}$

$x \cdot x \cdot x$

x^3

B) $\sqrt[4]{16x^4}$

$\sqrt[4]{16}$ $\quad$ $\sqrt[4]{x^4}$

$\boxed{\pm 2x}$

C) $\sqrt[3]{x^7} \cdot \sqrt[3]{x^2}$

$\sqrt[3]{x^7 \cdot x^2} = \sqrt[3]{x^9} = x^3$

$\sqrt[3]{x^7}$ $\quad$ $\sqrt[3]{x^2}$

$x^2 \sqrt[3]{x^1} \cdot 1\sqrt[3]{x^2}$

$x^2 \sqrt[3]{x^3}$

$x^2 \cdot x = x^3$

Try This!

a) $\sqrt[2]{50x^3}$

$\sqrt{25}\ \sqrt{2}$

$\sqrt{50} \quad \sqrt{x^3}$

$5\sqrt{2} \cdot x\sqrt{x}$

$\boxed{5x\sqrt{2x}}$

b) $\sqrt[4]{x^8} \bullet \sqrt[3]{x^4}$

$x^2 \cdot x\sqrt[3]{x}$

$x^3\sqrt[3]{x}$

c) $\dfrac{\sqrt[3]{x^5}}{4}$

$\dfrac{x\sqrt[3]{x^2}}{4}$

$$\sqrt[3]{\frac{x^5}{4}} = \frac{\sqrt[3]{x^5}}{\sqrt[3]{4}} = \frac{x^1\sqrt[3]{x^2}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$$

$$\frac{x\sqrt[3]{2x^2}}{\sqrt[3]{8}} = \frac{x\sqrt[3]{2x^2}}{2}$$

Rational Exponents

A **rational exponent** is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written by using rational exponents.

Rational Exponents

For any natural number n and integer m,

WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the nth root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the nth root raised to the mth power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Writing Expressions in Radical Form

Write the expression $(-32)^{\frac{3}{5}}$ in radical form and simplify.

$(-32)^{3/5}$ 3 ← power, 5 ← index

$\left(\sqrt[5]{-32}\right)^3 = (-2)^3 = \boxed{-8}$

Write the expression $64^{\frac{1}{3}}$ in radical form, and simplify.

$64^{1/3} = \left(\sqrt[3]{64}\right)^1 = (4)^1 = 4$

Try these!

Write the expression $4^{\frac{5}{2}}$ in radical form, and simplify.

$$4^{5/2} = \left(\sqrt{4}\right)^5 = 32$$

$$625^{3/4} = \left(\sqrt[4]{625}\right)^3 = (5)^3 = 125$$

Write the expression $625^{\frac{3}{4}}$ in radical form, and simplify.

Backwards

Write each expression by using rational exponents.

A. $\sqrt[8]{13^4}$

$13^{\frac{4}{8}} = 13^{1/2}$

B. $\sqrt[5]{13^{15}}$

$13^{\frac{15}{5}} = 13^{3}$

More Review?

Properties of Rational Exponents

For all nonzero real numbers *a* and *b* and rational numbers *m* and *n*,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplify each expression.

a) $7^{\frac{7}{9}} \bullet 7^{\frac{11}{9}}$ $\quad 7^{\frac{18}{9}} = 7^2 = 49$

$x^2 \cdot x^3$

b) $\dfrac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}}$ $\quad 16^{3/4 - 5/4} = 16^{-2/4} = 16^{-1/2} =$

$\dfrac{1}{16^{1/2}} = \dfrac{1}{\sqrt[2]{16}} = \boxed{\dfrac{1}{4}}$