Operations with Functions

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Today's Objectives:

~ We will learn to add, subtract, multiply and divide functions.

~We will learn to compose and write and evaluate composite functions

Function Operations

You can perform operations on functions in the same way that you perform operations on numbers.

Notation for Function Operations	
Operation	**Notation**
Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Dive Right In!

Given $f(x) = 4x^2 + 3x - 1$ and $g(x) = 6x + 2$, find each function.

a) $(f + g)(x) = 4x^2 + 9x + 1$

$f(x) + g(x)$

$4x^2 + 3x - 1 + 6x + 2$

b) $(f - g)(x) = 4x^2 - 3x - 3$

$f(x) - g(x)$

$(4x^2 + 3x - 1) - (6x + 2)$

$4x^2 + 3x - 1 - 6x - 2$

Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.

a) $(f + g)(x) = x^2$

$5x - 6 + x^2 - 5x + 6$

b) $(f - g)(x) = -x^2 + 10x - 12$

$5x - 6 - x^2 + 5x - 6$

Multiply and Dividing

Be Careful! When dividing functions, be sure to list any domain restrictions.

Given $f(x) = 6x^2 - x - 12$ and $g(x) = 2x - 3$, find each function.

a) $(fg)(x)$

$(6x^2 - x - 12)(2x - 3)$

$12x^3 - 18x^2 - 2x^2 + 3x - 24x + 36$

$12x^3 - 20x^2 - 21x + 36$

b) $\left(\frac{f}{g}\right)(x)$

$2x - 3 = 0$

$2x = 3$

$x \neq 3/2$

$\frac{6x^2 - x - 12}{2x - 3}$

$\frac{(2x-3)(3x+4)}{(2x-3)} =$

$3x + 4 \quad x \neq \frac{3}{2}$

Given $f(x) = x + 2$ and $g(x) = x^2 - 4$, find each function.

a) $(fg)(x)$

$(x+2)(x^2-4)$

$x^3 + 2x^2 - 4x - 8$

b) $\left(\frac{g}{f}\right)(x)$

$\frac{x^2 - 4}{x + 2}$

$\frac{(x+2)(x-2)}{(x+2)} =$

$x \neq -2$

$x - 2$

Level Up!

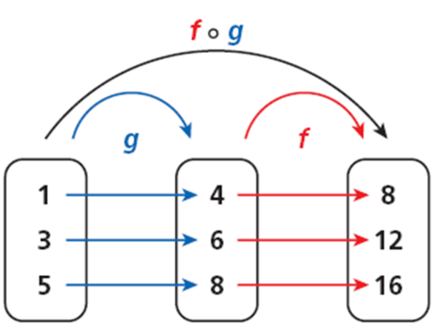
Another function operation uses the output from one function as the input for a second function. This operation is called the **composition of functions**.

Composition of Functions

The composition of functions f and g is notated

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)(x)$ is all values of x in the domain of g such that $g(x)$ is in the domain of f.



To find $(f \circ g)(1)$, first find $g(1)$.

$g(1) = 4$

Then use 4 as the input into f:

$f(4) = 8$

So $(f \circ g)(1) = f(g(1)) = 8$.

Examples!

Given $f(x) = 2^x$ and $g(x) = 7 - x$, find each value.

a) $f(g(4))$

$g(4) = 7 - 4 = 3$

$g(4) = 3$

$f(3) = 2^3 = 8$

$f(g(4)) = 8$

b) $g(f(4))$

$f(4) = 2^4 = 16$

$g(16) = 7 - 16 = -9$

$g(f(4)) = -9$

Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.

a) $f(g(3)) = 15$

$g(3) = 9$

$f(9) = 15$

b) $g(f(3)) = 9$

$f(3) = 3$

$g(3) = 9$

Composition of Functions

Given $f(x) = x^2 - 1$ and $g(x) = \frac{x}{1-x}$, write each composite function. State the domain of each.

a) $g(f(x))$

$g(x^2-1) = \frac{(x^2-1)}{1-(x^2-1)} = \frac{x^2-1}{1-x^2+1} =$

$\frac{x^2-1}{-x^2+2}$

b) $g(f(x))$

Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite. State the domain of each.

a) $f(g(x))$

$f(\sqrt{x}+2)$

$f(\sqrt{x}+2) = 3(\sqrt{x}+2) - 4$

$f(\sqrt{x}+2) = 3\sqrt{x} + 6 - 4$

$f(\sqrt{x}+2) = 3\sqrt{x} + 2$

b) $g(f(x))$

$g(3x-4)$

$g(3x-4) = \sqrt{3x-4} + 2$

Word Problems!

Jake imports furniture from Mexico. The exchange rate is 11.30 pesos per U.S. dollar. The cost of each piece of furniture is given in pesos. The total cost of each piece of furniture includes a 15% service charge.

A. Write a composite function to represent the total cost of a piece of furniture in dollars if the cost of the item is *c* pesos.

B. Find the total cost of a table in dollars if it costs 1800 pesos.

Why Do We Even Care?

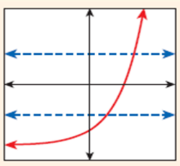
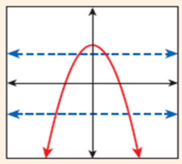
Inverse Functions!

The inverse of a function $f(x)$ "undoes" $f(x)$. Its graph is a reflection across line $y = x$. *The inverse may or not be a function*.

How do we know if the inverse of a function is a function?

Recall that the vertical-line test can help you determine whether a relation is a function. Similarly, the *horizontal-line* test can help you determine whether the inverse of a function is a function.

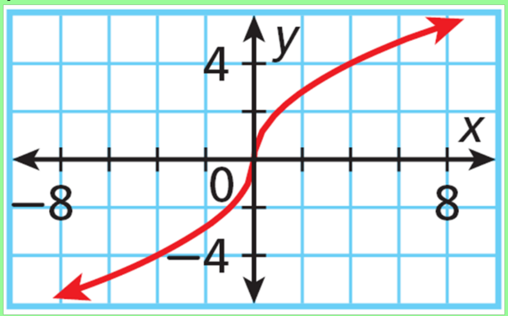
Horizontal-line Test

WORDS	EXAMPLES	
If any horizontal line passes through more than one point on the graph of a relation, the inverse relation is not a function.	 Inverse is a function.	 Inverse is not a function.

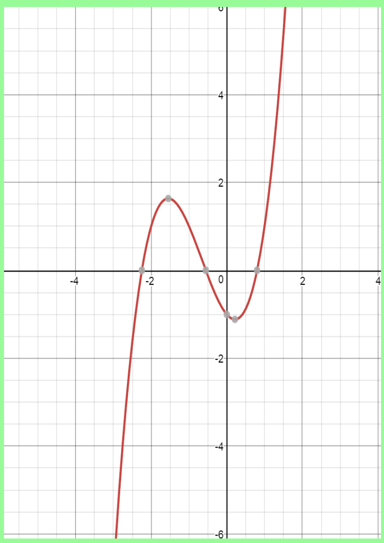
Horizontal Line Test

Use the horizontal-line test to determine whether the inverse of each relation is a function.

a)



b)



How Do We Find the Inverse of a Function?

Simple! Switch x and y. Then, solve!

Find the inverse of $f(x) = \sqrt[3]{x+1}$. Determine whether it is a function, and state its domain and range.

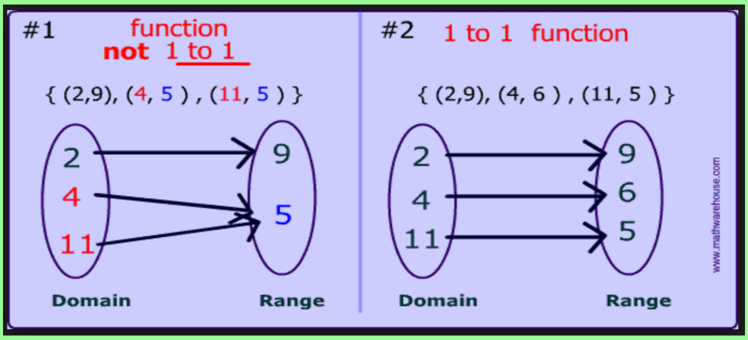
Find the inverse of $f(x) = x^3 - 2$. Determine whether it is a function, and state its domain and range.

Try This!

Find the inverse of $f(x)=\frac{5x+9}{6}$. Determine whether it is a function, and state its domain and range.

More on Inverses

You have seen that the inverses of functions are not necessarily functions. When both a relation and its inverses are functions, the relation is called a *one-to-one function*. In a **one-to-one function**, each y-value is paired with exactly one x-value.



Where it All Leads To

You can use composition of functions to verify that two functions are inverses. Because inverse functions "undo" each other, when you compose two inverses the result is the input value *x*.

Identifying Inverse Functions

WORDS	ALGEBRA	EXAMPLE
If the compositions of two functions equal the input value, the functions are inverses.	If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.	$f(x) = 3x$ and $g(x) = \frac{1}{3}x$ $f(g(x)) = 3\left(\frac{1}{3}x\right) = x$ $g(f(x)) = \frac{1}{3}(3x) = x$

Determine by composition whether each pair of functions are inverses.

$f(x) = 3x - 1$ and $g(x) = \frac{1}{3}x + 1$

You Try!

a) **For $x \neq 1$ or 0, $f(x) = \frac{1}{x - 1}$ and $g(x) = \frac{1}{x} + 1$.**

b) $f(x) = \frac{2}{3}x + 6$ and $g(x) = \frac{3}{2}x - 9$