Solving Quadratics

Quick Review.... You Tell Me

Factoring:

$$x^2 - 16x + 63$$

$(x-7)(x-9) \rightarrow$ Factor

$x-7=0 \quad x-9=0$

$x=7 \quad x=9 \rightarrow$ Solve

$$7x^2 - 31x - 20$$

	x	-5
$7x$	$7x^2$	$-35x$
4	$4x$	-20

$-140x^2$

↑ mult.

$-31x$

↑ add

$(7x+4)(x-5)$

$x=-4/7 \quad x=5$

$$2b^2 + 17b + 21$$

$2b^2 \cdot 21 = 42b^2$

$42b^2$

↑ mult.

$17b$

↑ add

$\left.\right\rangle$ $14b$, $3b$

$(2b^2+14b)+(3b+21)$

$2b(b+7)+3(b+7)$

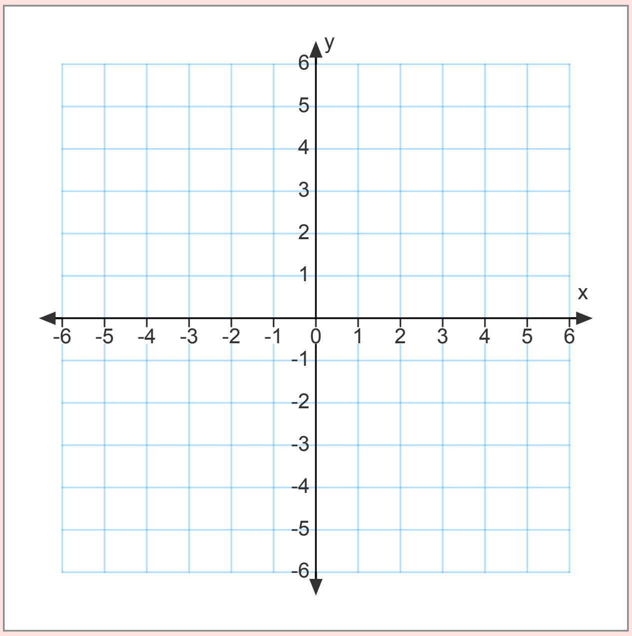
$(b+7)(2b+3)$

$b=-7 \quad b=-\frac{3}{2}$

Graphing...

Find the zeros and graph the function by hand

$$g(x) = x^2 - 22x + 121$$



Using the calculator:

$$g(x) = x^2 - 8x - 20$$

$$y = x^2 - 8x - 20$$

Quadratic Formula:

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Discriminant:

$$b^2-4ac$$

positive: 2 real solutions

0 = 1 real solution

negative: 2 imaginary

$$g(x) = -x^2 - x - 1$$

$$(-1)^2-4(-1)(-1) = -3 \rightarrow 2 \text{ imaginary}$$

$$\frac{-(-1) \pm \sqrt{(-1)^2-4(-1)(-1)}}{2(-1)} = \frac{1 \pm \sqrt{-3}}{-2}$$

$\sqrt{-1} \cdot \sqrt{3}$

$$\frac{1 \pm i\sqrt{3}}{-2}$$

$$\frac{1+i\sqrt{3}}{-2}$$

$$\frac{1-i\sqrt{3}}{-2}$$

Today's Warm Up:

Solve the following using square roots.

$$4x^2 - 20 = 5$$

$$+20 \quad +20$$

$$\frac{4x^2}{4} = \frac{25}{4}$$

$$\sqrt{x^2} = \sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}}$$

$$x = \pm\frac{5}{2}$$

$$2(x+2)^2 - 5 = 8$$

$$+5 \quad +5$$

$$\frac{2(x+2)^2}{2} = \frac{13}{2}$$

$$\sqrt{(x+2)^2} = \sqrt{\frac{13}{2}}$$

$$x+2 = \pm\sqrt{\frac{13}{2}} = \frac{\sqrt{13}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$$

$$x+2 = \pm\frac{\sqrt{26}}{2}$$

$$x+2 = \frac{\sqrt{26}}{2} \qquad x+2 = \frac{-\sqrt{26}}{2}$$

$$-2 \qquad\qquad -2$$

$$x = -2 + \frac{\sqrt{26}}{2} \qquad x = -2 - \frac{\sqrt{26}}{2}$$

$$\frac{1}{5}x^2 + 2 = \frac{3}{5}x^2$$

$$-\frac{1}{5}x^2 \qquad -\frac{1}{5}x^2$$

$$\left(\frac{5}{2}\right)2 = \frac{2}{5}x^2\left(\frac{5}{2}\right)$$

$$\sqrt{5} = \sqrt{x^2}$$

$$\pm\sqrt{5} = x$$

Do Now:

Factor the following:

$$9x^2 + 12x + 4$$

$9x^2 \cdot 4 = 36x^2$

$\uparrow$

x

$12x \rightarrow +$

$(9x^2 + 6x) + (6x + 4)$

$3x(3x+2) + 2(3x+2)$

$(3x+2)(3x+2)$

$\sqrt{(3x+2)^2} = \sqrt{16}$

$3x+2 = \pm 4$

$3x+2 = 4$ $3x+2 = -4$

$-2 \; -2$ $-2 \; -2$

$\frac{3x}{3} = \frac{2}{3}$ $3x = -6$

$x = 2/3$ $x = -2$

Completing the Square!

You can use algebra to rewrite any quadratic expression as a perfect square.

If a quadratic expression of the form $x^2 + bx$ *cannot* model a square, you can add a term to form a perfect square trinomial. This is called **completing the square**.

Completing the Square

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.	$x^2 + 6x + \blacksquare$ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx + \blacksquare$ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

Some Examples

Complete the square for the expression. Write the resulting expression as a binomial squared.

$x^2 + \underbrace{4}x + \blacksquare$

$\left(\frac{b}{2}\right)^2$

$b=4$

$\frac{4}{2} = (2)^2 = 4$

x^2+4x+4

$(x+2)^2$

$x^2 + 3x + \blacksquare$

$b=3$

$\frac{3}{2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2+3x+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2$

Studious Steps

Solving Quadratic Equations $ax^2 + bx + c = 0$ by Completing the Square

1. Collect variable terms on one side of the equation and constants on the other.
2. As needed, divide both sides by a to make the coefficient of the x^2-term 1.
3. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
4. Factor the variable expression as a perfect square.
5. Take the square root of both sides of the equation.
6. Solve for the values of the variable.

Some Practice

Solve the equation by completing the square.

$\boldsymbol{x^2 \underbrace{- 2}_{c} = 9x}$

① "c" to the other side (isolate)

② Check leading coefficient is 1

③ "Complete the square"

④ Factor

⑤ Solve

$x^2 = 9x + 2$

$-9x \quad -9x$

$x^2 - 9x = 2$

$\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

$x^2 - 9x + \frac{81}{4} = 2 + \frac{81}{4}$

$\frac{8}{4} + \frac{81}{4}$

$x^2 - 9x + \frac{81}{4} = \frac{89}{4}$

$\left(x - \frac{9}{2}\right)^2 = \frac{89}{4}$

$\sqrt{\left(x - \frac{9}{2}\right)^2} = \pm\sqrt{\frac{89}{4}}$

$x - \frac{9}{2} = \pm\frac{\sqrt{89}}{2}$

$x - \frac{9}{2} = \frac{\sqrt{89}}{2}$ $\qquad$ $x - \frac{9}{2} = -\frac{\sqrt{89}}{2}$

$+\frac{9}{2} \quad +\frac{9}{2}$

$x = \frac{9}{2} + \frac{\sqrt{89}}{2}$ $\qquad$ $x = \frac{9}{2} - \frac{\sqrt{89}}{2}$

$\boldsymbol{\frac{3x^2}{3} - \frac{24x}{3} = \frac{27}{3}}$

$x^2 - 8x = 9$

$\left(\frac{-8}{2}\right) = (-4)^2 = 16$

$x^2 - 8x + 16 = 9 + 16$

$x^2 - 8x + 16 = 25$

$\sqrt{(x-4)^2} = \sqrt{25}$

$x - 4 = \pm 5$

$x - 4 = 5$ $\qquad$ $x - 4 = -5$

$x = 9$ $\qquad$ $x = -1$

Try this!

$$\frac{3x^2}{3} + \frac{6x}{3} = \frac{1}{3}$$

$$x^2 + 2x = 1/3$$

$$\frac{2}{2} = (1)^2 = 1$$

$$x^2 + 2x + 1 = 1 + 1/3$$

$$x^2 + 2x + 1 = 4/3$$

$$\sqrt{(x+1)^2} = \sqrt{4/3}$$

$$x + 1 = \pm \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x + 1 = \pm \frac{2\sqrt{3}}{3}$$

$$-1 \quad -1$$

$$x = -1 \pm \frac{2\sqrt{3}}{3}$$

Do Now Answers:

1) 121

$(x-11)^2$

2) 81/4

$(x+\frac{9}{2})^2$

3) $x=-7\pm\sqrt{73}$

4) $x=2\pm\sqrt{3}$

5) $x=-1,4$

6) $x=-4\pm 2\sqrt{3}$

(4) $\frac{2x^2}{2}-\frac{8x}{2}=\frac{-2}{2}$

$x^2-4x=-1$

$\frac{-4}{2}=(-2)^2=4$

$x^2-4x+4=-1+4$

$\sqrt{(x-2)^2}=\sqrt{3}$

$x-2=\pm\sqrt{3}$

$x=2\pm\sqrt{3}$

$2+\sqrt{3}\quad 2-\sqrt{3}$

(5) $x^2=3x+4$

$-3x\quad -3x$

$x^2-3x=4$

$\frac{-3}{2}=\left(\frac{-3}{2}\right)^2=\frac{9}{4}$

$x^2-3x+\frac{9}{4}=4+\frac{9}{4}$

$\frac{16}{4}+\frac{9}{4}$

$x^2-3x+\frac{9}{4}=\frac{25}{4}$

$\sqrt{\left(x-\frac{3}{2}\right)^2}=\sqrt{\frac{25}{4}}$

$x-\frac{3}{2}=\pm\frac{5}{2}$

$+\frac{3}{2}\quad +\frac{3}{2}$

$\frac{3}{2}+\frac{5}{2}=\frac{8}{2}=4$

$\frac{3}{2}-\frac{5}{2}=\frac{-2}{2}=-1$

Remember...

Before, we could convert from vertex form to standard form by expanding. Now we can convert from standard form to vertex form by completing the square.

$y=(x+1)^2-3$

$y=(x+1)(x+1)-3$

$y=x^2+2x+1-3$

$y=x^2+2x-2$

$f(x) = x^2 + 24x + 145$

$y = x^2+24x+145$

$-145 \qquad -145$

$y-145 = x^2+\underline{24}x$

$\frac{24}{2}=(12)^2=144$

$y-145+144 = x^2+24x+144$

$y-1=(x+12)^2$

$+1 \qquad +1$

$\boxed{y=(x+12)^2+1} \rightarrow f(x)=(x+12)^2+1$

$(-12,1)$

Your Turn

$g(x) = 5x^2 - 50x + 128$

$y = 5x^2 - 50x + 128$
$-128 \qquad\qquad -128$

$y - 128 = 5x^2 - 50x$

$y - 128 = 5(x^2 - 10x)$

$\frac{-10}{2} = (-5)^2 = 25$

$y - 128 + 125 = 5(x^2 - 10x + 25)$

$y - 3 = 5(x-5)^2$
$+3 \qquad +3$

$y = 5(x-5)^2 + 3$

$(5, 3)$

$$g(x) = 9x^2 + 18x - 1$$

$y + 1 = 9x^2 + 18x$

$y + 1 = 9(x^2 + 2x)$

$\frac{2}{2} = (1)^2 = 1$

$y + 1 + 9 = 9(x^2 + 2x + 1)$

$y + 10 = 9(x + 1)^2$

$-10 \qquad -10$

$y = 9(x+1)^2 - 10$

$(-1, -10)$

So why is completing the square so important?

$$ax^2+bx+c=0$$
$$\quad -c \quad -c$$

$$\frac{ax^2}{a}+\frac{bx}{a}=\frac{-c}{a}$$

$$x^2+2x=1$$
$$\frac{2}{2}=(1)^2=1$$

$$x^2+\frac{b}{a}x=\frac{-c}{a}$$

$$\frac{\frac{b}{a}}{2}=\frac{b}{a}\cdot\frac{1}{2}=\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{-c}{a}\cdot\frac{4a}{4a}+\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{b^2-4ac}{4a^2}$$

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$
$$-\frac{b}{2a} \quad -\frac{b}{2a}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Word Problems

A 9 by 12 rectangular picture is framed by a border of uniform width. Given that the combined area of the picture and the frame is 180 square units, what is the width of the border?

The general function that approximates the height of a projectile on Earth after t seconds is given by:

$$h(t) = -16t^2 + v_0 t + h_0$$

where -16 ft/s^2 is the constant due to Earth's gravity, v_o is the initial vertical velocity in ft/s (at t = 0), and h_o is the initial height in ft (at t = 0).

A football is kicked from the ground with an initial velocity of 48 ft/s.How many seconds before it hits the ground?

v_o = initial velocity = 48

h_o = initial height = 0

$h(t) = -16t^2 + 48t + 0$

$h(t) = -16t^2 + 48t$

$0 = -16t^2 + 48t$

$t = 0$ $t = 3$